DEVELOPING MATHEMATICAL MODELS FOR PRELIMINARY INTERNAL CONTROL EVALUATIONS OF INVENTORY SYSTEMS IN AUDITING

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THESIS

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OF INVENTORY SYSTEMS IN AUDITING

by

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Russ Richards

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In an article entitled "Internal Control Evaluation: How The Computer Can Help" David C. Burns and James K. Loebbecke presented a computer simulation audit approach for evaluating internal control. In their article Burns and Loebbecke applied their simulation approach to evaluate an illustrative manufacturing inventory internal control system. This thesis presents two alternative mathematical approaches for solving the Burns/Loebbecke inventory problem: The two mathematical approaches presented are

compared and the unique conditions necessary for the application of each approach are discussed.

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In an article entitled "Internal Control Evaluation: How The Computer Can Help" David C. Burns and James K. Loebbecke presented a computer simulation audit approach for evaluating internal control. In their article Burns and Loebbecke applied their simulation approach to evaluate an illustrative manufacturing inventory internal control system. This thesis presents two alternative mathematical approaches for solving the Burns/Loebbecke inventory problem: The two mathematical approaches presented are compared and the unique conditions necessary for the application of each approach are discussed.

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I. INTRODUCTION

A. GENERAL

Given the premise that the volume of transactions processed by most entities precludes a detailed audit, the question arises as to the degree to which the auditor should be permitted to restrict his detailed tests by relying on the system of internal control.

The second generally accepted auditing standard of field work states that the independent auditor is to study and evaluate his client's system of internal control "as a basis for reliance thereon" and for determining the timing and the extent of testing to be performed under the circumstances. Hence, the primary objective of internal control evaluation is to ascertain the extent to which the internal control system may be relied upon and thus establish a basis for determining the nature, timing and extent of detailed auditing procedures.

A related, but secondary objective is to provide sufficient knowledge of a client's affairs to make timely suggestions for not only strengthening the system of internal control but, more importantly, for increasing it's efficiency and effectiveness.

Statement on Auditing Procedure No. 54 (henceforth SAP 54)[1] provides practitioners the following guidance for performing a study and evaluation of internal control:

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[1] Statement on Auditing Procedure No. 54: "The Auditor's study and evaluation of internal control" issued by the Committee on Auditing procedure of the American Institute of Certified Public Accountants, November 1972. The Journal of Accountancy, March 1973, page 62.

"A conceptually logical approach to the auditor's evaluation of accounting control, which focuses directly on the purpose of preventing or detecting material errors and irregularities in financial statements, is to apply the following steps in considering each significant class of transactions and related assets involved in the audit;

a. Consider the types of errors and irregularities that could occur.

b. Determine the accounting control procedures that should prevent or detect such errors and irregularities.

c. Determine whether the necessary procedures are prescribed and are being followed satisfactorily.

d. Evaluate any weaknesses - i.e., types of potential errors and irregularities not covered by existing control procedures - to determine their effect on (1) the nature, timing or extent of auditing procedures to be applied and, (2) suggestions to be made to the client."

Four basic types of internal control work are normally performed by practicing auditors to comply with guidance provided by SAP 54.[2]

1. A study and review of prescribed internal control.
2. A "preliminary" or conceptual evaluation of prescribed internal controls.
3. Compliance tests of selected internal controls.
4. A "final" evaluation of internal control.

This thesis focuses on the procedures required to accomplish a preliminary evaluation of internal control.

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[2] David C. Burns and James K. Loebbecke: "Internal Control Evaluation: How The Computer Can Help". The Journal of Accountancy, August, 1975.

B. PRELIMINARY EVALUATION OF INTERNAL CONTROL[3]

The preliminary evaluation of internal control plays a key role in the audit process when the SAP 54 (statement on Auditing Procedures No. 54) approach is employed.

The primary purpose of the auditor's preliminary evaluation is to obtain a rational basis for formulating a tentative decision concerning the degree of reliance to place upon the various facets of the client's prescribed system of internal controls.

However, the benefits which should be derived from this preliminary evaluation extend far beyond this primary prupose. A careful thorough preliminary evaluation helps assure a valid tentative audit strategy of reliance upon internal control, compliance tests, and substantive procedures.

The planning of such a strategy at the earliest stages of the audit can save both the auditor and his client much time and money in the long run. A careless preliminary evaluation can cause the auditor to plan an inappropriate audit strategy based on incorrect assessment of the adequacy of internal control. This type of erroneous tentative strategy can lead to unnecessary compliance tests, poorly designed substantive test, and expensive emergency procedures. Hence the preliminary evaluation is a very important step in the audit process.

C. PURPOSE

In an article entitled "Internal Control Evaluation: How The

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[3] David C. Burns: "Extending the study and evaluation of internal controls to meet system complexitites", The CPA Journal, May, 1974, pp. 33 - 34.

Computer Can Help" David C. Burns and James K. Loebbecke explained how the Computer can serve the auditor as a preliminary evaluation audit tool. To illustrate this potential, Burns and Loebbecke applied simulation to a preliminary evaluation problem that involved an inventory accounting and internal control system of a moderately complex manufacturing firm.

The purpose of this thesis is to demonstrate the extent to which the illustrative problem proposed by Burns and Loebbecke can be solved by two different analytical methods. A mean value analytical method is demonstrated in Chapter V. A stochastic method is demonstrated in Chapter VI.

II. INTRODUCTION OF THE ILLUSTRATIVE PROBLEM

As was previously mentioned the illustrative problem proposed by Burns and Loebbecke involves a raw materials inventory accounting and internal control system of a manufacturing firm. This raw materials system is a facet of a total manufacturing inventory system proposed by Burns.[4] A summary description of Burn's illustrative problem appears in Appendix A of this thesis.

A brief summary description of the raw materials facet of the problem is presented in the following section for the reader's conveience.

A. SUMMARY OF THE BURNS AND LOEBBECKE PROBLEM

This problem requires that tolerable compliance levels be established for a raw materials inventory accounting and internal control subsystem. The problem involves a manual raw materials inventory accounting and internal control subsystem which is flowcharted in Figure I.

It is the company's policy to carry all inventories at predetermined standard cost. A physical inventory is conducted each year by management on September 30. Both the perpetual stock control records and the financial accounts are adjusted to agree with this September 30th physical inventory. The company relies upon its system of internal control to assure accurate financial inventory account balances at the December 31 year end. Budgeted inventory activity for the three month

[4] This article deals with the problem proposed in Professor Burn's doctoral dissertation "Audit Evidence Evaluation Using Computer Simulation with Special Emphasis on Ascertaining the Reliability of Accounting Data", Indiana University, Graduate School of Business, 1971.

period to end December 31 of the current year and the inventory accounting and internal control subsystem (flowcharted) are presented in Table I and Figure I respectively.

B. PRELIMINARY EVALUATION ASSUMPTIONS

The company's independent auditor has completed his review and visual inspection of prescribed raw material controls on May 15 of the current year. He is currently in the process of performing his preliminary evaluation of the inventory system.

In the course of performing his preliminary evaluation, the auditor has identified all inventory controls which seem pertinent to his preliminary evaluation at this early stage of the audit.

He has used his professional impressions to obtain the potential frequency rates and error magnitudes for many types of errors. These are illustrated in Table II.

Having established the facts and assumptions presented above, the auditor must now turn to the difficult task of performing a preliminary evaluation of the inventory system.

This preliminary evaluation will provide him a basis for planning his physical inventory strategy. i.e., by the interpretation of the results of preliminary internal control evaluation, the auditor can decide, on a tentative basis, whether or not the system seems strong enough to support heavy reliance upon a September 30th physical inventory for audit purposes.

If the auditor decides that the subsystem seems strong enough to support heavy audit reliance upon the September 30 physical inventory, he must further plan compliance tests, establish tolerable levels of compliance and design substantive tests for interim inventory transactions.

In determining tolerable compliance levels, the auditor must decide what levels of compliance with pertinent inventory controls seem necessary to justify reliance upon the subsystem for the three month interim period to end December 31 of the current year.

III. SUMMARY OF PERTINENT FACTS AND PRELIMINARY EVALUATION ASSUMPTIONS

The following diagrams contain the facts and preliminary evaluation assumptions related to one of four types of raw materials included in the Burns/Loebbecke illustration:

Raw Material # 1

Purchase 40,000 units (budgeted). Shipment μ = 200 σ = 25	33.600 units requisition production order μ = 150 σ = 35
error factors	error factors
Quantity error 25% chance to overstate by 10%	Quantity error 15% chance to overstate by 10% by Dept. # 1.
Costing error 10% chance to apply erroneous unit price STD price $ 13.5/unit Erroneous $ 6.5/unit	Costing error 10% chance to apply erroneous unit price

W. I. P. Product 1. Dept. # 1

Requisitioned 33.600 units (Budgeted)	33.600 units transferred to Dept. # 2.
error factors	error factors

Production count quantity error may occur in Dept. # 1. If a count error occurs in Dept. 1 and escapes the detection of Dept. 1 controls it moves to Dept. 2. Labor and overhead will be charged by cost accounting Dept. (illusion figures).

W. I. P. Product 1. Dept. # 1

Labor Hr.: 8% chance to apply erroneous unit STD Hr. STD Labor Hr.: 0.06 hr/unit erroneous Hr.: 0.04	
Labor Rate: 10% chance to apply erroneous unit labor rates. Type I and Type II given error there are 50-50 chance to apply Type I and Type II. STD Labor Rate: 6.2/hr. erroneous Type I: 6.0 Type II: 5.6	All undetected (debit) will be transferred to Dept. # 2.
Overhead Rate: 8% chance to apply erroneous overhead rate. STD Rate $ 12.85/labor hr. erroneous $ 11.4/labor hr.	

W. I. P. Product 1. Dept # 2

transferred from Dept. # 1. 33.600 units lot size 150	will transfer 30,280 units to finished good inventory
Quantity error Overstatements of production counts also occur in Dept. 2 Their frequency rate is .08 and their magnatude is 5% overstatement. Dept. 2 count error can occur on an order previously overstated in Dept. 1.	No additional quantity errors occur on orders transferred to finished goods. However, all undetected production count errors pass through to finished goods.
Labor Hr.: No error in Product # 1. But there are errors in other products. STD 0.04 hr/unit	

W. I. P. Product 1. Dept. # 2
(continued)

Labor Rate: 10% chance to apply erroneous unit cost. Also Type I. Type II. STD: \$ 5.6/labor hr. erroneous Type I 5.4/Hr. Type II 6.2/Hr. Overhead Rate: 8% chance to apply erroneous overhead rate. STD: \$ 51.55/labor hr. erroneous: \$ 44.05/labor hr.	

Finished Good # 1

transferred → 30.280 units (Budgeted). Error Factors Quantity error 15% chance to overstate by 10% by Dept. # 1. 8% chance to overstate by 5% by Dept. # 2. Costing error 8% chance to apply erroneous unit cost. STD unit cost: \$ 16.929/unit erroneous cost: \$ 9.19/unit	sale → no error

On the basis of the above assumptions concerning accounting error flows, error rates and error magnitudes the auditor in the Burn's and Loebbecke illustration must somehow assess potential financial statement consequences of the system in dollar terms. This requires the auditor to determine statistic means and standard deviations for the dollar errors at each stage of the accounting process. These error statistics must further

be combined to arrive at grand error statistics to determine potential financial statement consequences.

This task was performed by Burn's and Loebbecke via computer simulation.

The development of a rigorous mathematical model of internal control is beyond the intended scope of this thesis. This thesis effort is directed toward the development of an alternative approximation model. A model, simple enough that it might be developed by an audit practitioner.

The formulation of such an alternative model requires an understanding of the behavior of the assumed accounting system and the flow of the accounting data. The following chapters demonstrate how this can be done.

TABLE I

BUDGET OF RAW MATERIALS ACTIVITY

FOR THE FUTURE THREE-MONTH PERIOD TO END DECEMBER 31

	UNITS	DOLLARS
BUDGETED BEGINNING INVENTORY (SEPT. 30)	29,500	339,100
BUDGETED PURCHASES FOR THE PERIOD	140,000	1,584,800
SUBTOTAL	169,500	$1,923,900
BUDGETED USAGE	128,500	1,450,400
BUDGETED ENDING INVENTORY (DEC. 31)	41,000	$ 473,500

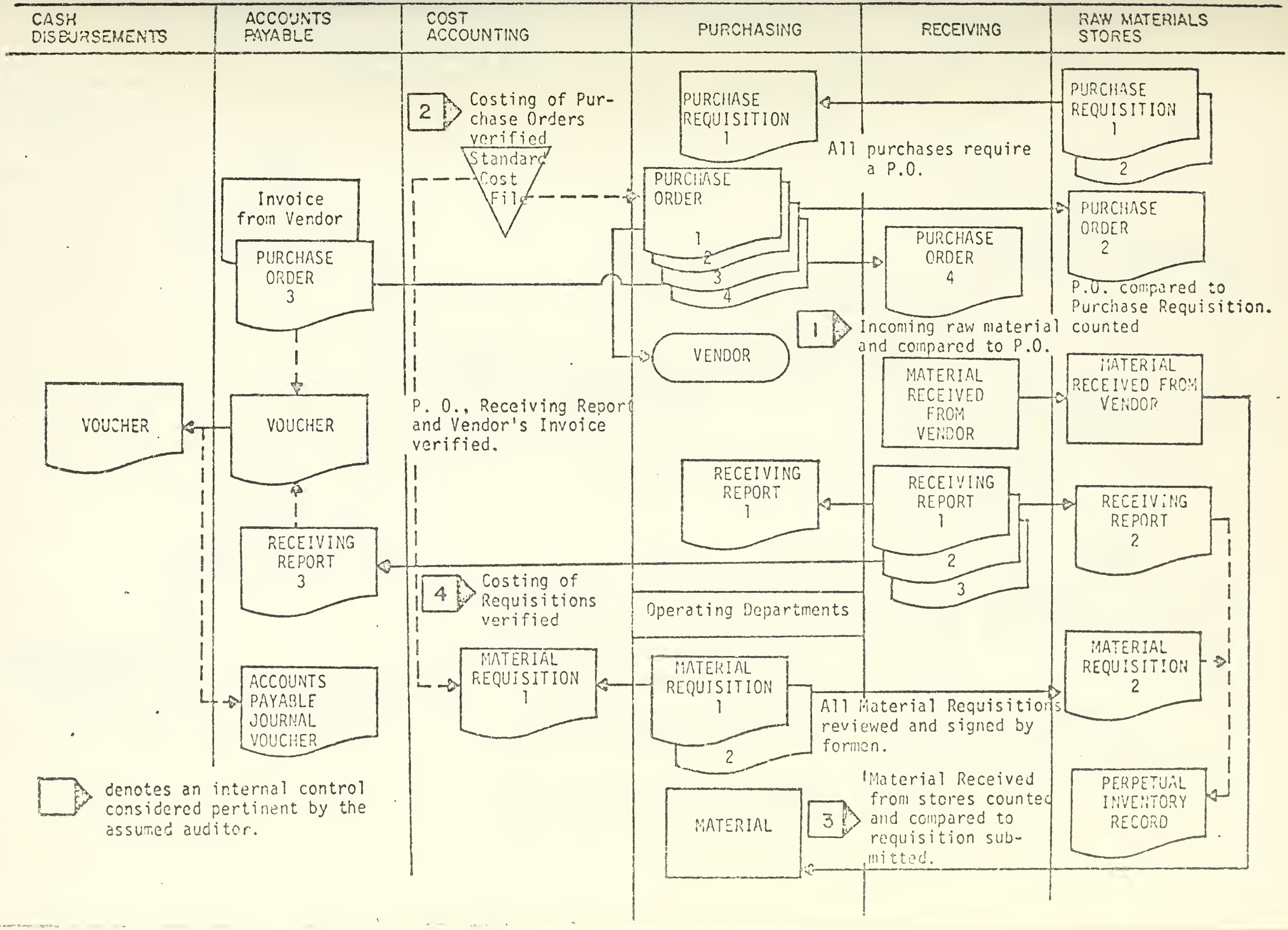
TABLE II

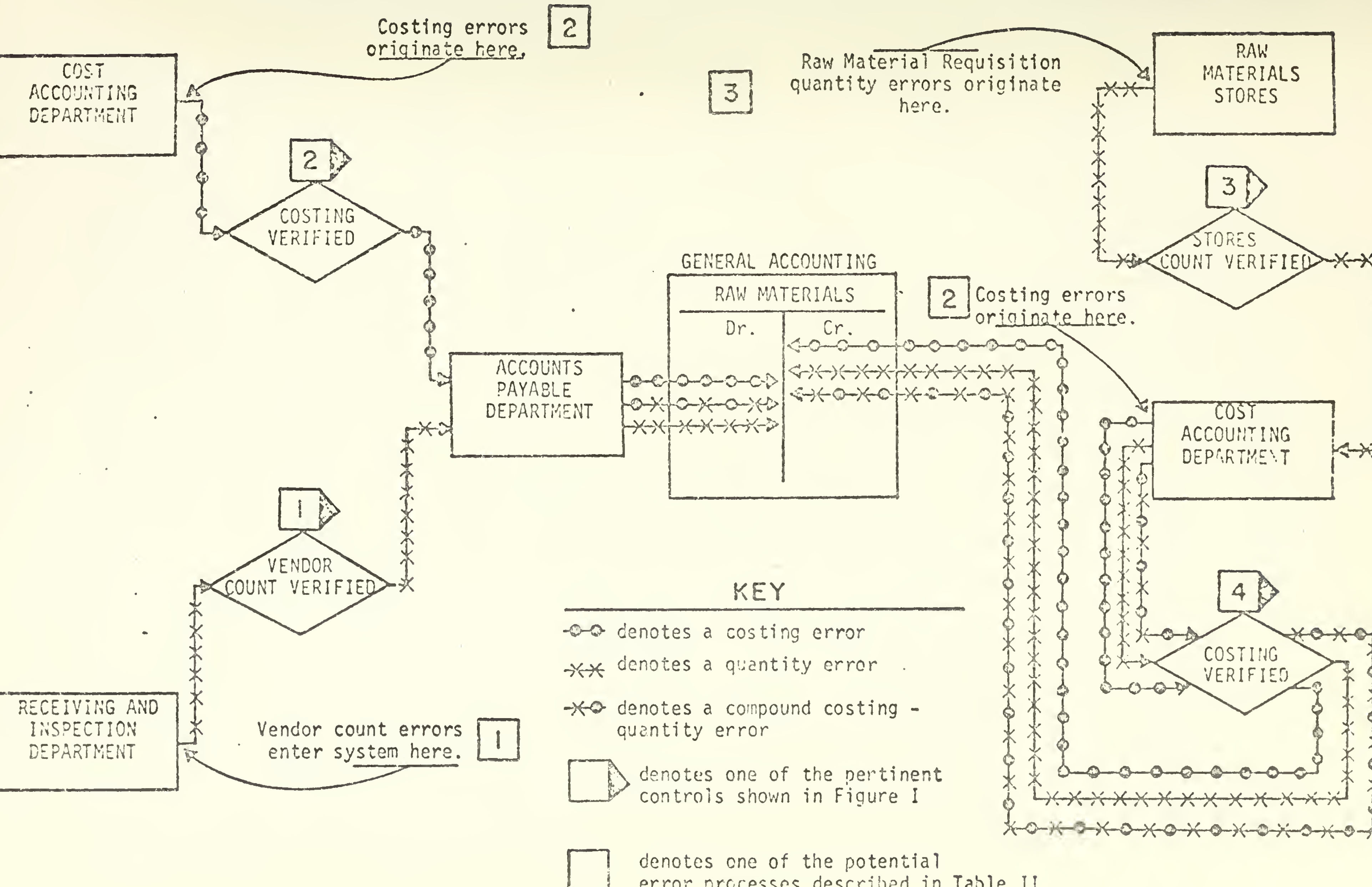
QUANTITATIVE DESCRIPTIONS OF POTENTIAL ERRORS

e of Error	Symbols*	Potential Frequency Rate**	Potential Error Magnitude**
dor Count Errors	1	25 out of every 100 counts or 25%	10% understatement of true quantity
ting Errors	2	10 out of every 100 transactions or 10%	Random application of an inappropriate standard price. (See Figure III for further details)
Material uisition nt Errors	3	15 out of every 100 requisitions or 15%	10% overstatement of true quantity.

mbols in this column are used to identify types of potential errors in other gures presented later in this article.

equency rates and Magnitudes could be described in either a "most pessimistic" "most likely" manner depending on the preference of the auditor involved.


CASH DISBURSEMENTS
ACCOUNTS PAYABLE
COST ACCOUNTING
PURCHASING
RECEIVING
RAW MATERIALS STORES
PURCHASE REQUISITION 1
2
PURCHASE REQUISITION 1
All purchases require a P.O.
2
Costing of Purchase Orders verified
Standard Cost File
PURCHASE ORDER 1
2
3
4
PURCHASE ORDER 2
P.O. compared to Purchase Requisition. counted
PURCHASE ORDER 4
Invoice from Vendor
PURCHASE ORDER 3
1
Incoming raw material and compared to P.O.
VENDOR
MATERIAL RECEIVED FROM VENDOR
MATERIAL RECEIVED FROM VENDOR
VOUCHER
VOUCHER
P. O., Receiving Report and Vendor's Invoice verified.
RECEIVING REPORT 1
RECEIVING REPORT 1
2
3
RECEIVING REPORT 2
RECEIVING REPORT 3
4
Costing of Requisitions verified
Operating Departments
MATERIAL REQUISITION 1
MATERIAL REQUISITION 1
2
MATERIAL REQUISITION 2
All Material Requisitions reviewed and signed by formen.
ACCOUNTS PAYABLE JOURNAL VOUCHER
denotes an internal control considered pertinent by the assumed auditor.
MATERIAL
3
Material Received from stores counted and compared to requisition submitted.
PERPETUAL INVENTORY RECORD


COST ACCOUNTING DEPARTMENT
Costing errors originate here.
2
COSTING VERIFIED
3
Raw Material Requisition quantity errors originate here.
RAW MATERIALS STORES
STORES COUNT VERIFIED
GENERAL ACCOUNTING
RAW MATERIALS
Dr.
Cr.
Costing errors originate here.
ACCOUNTS PAYABLE DEPARTMENT
COST ACCOUNTING DEPARTMENT
1
VENDOR COUNT VERIFIED
4
RECEIVING AND INSPECTION DEPARTMENT
Vendor count errors enter system here.
KEY
denotes a costing error
denotes a quantity error
denotes a compound costing - quantity error
denotes one of the pertinent controls shown in Figure I
denotes one of the potential error processes described in Table II

IV. METHODS OF ATTACK

To solve this problem an understanding of the system and accounting flow is a prerequisite because relative magnitudes of error effects differ according to the assumed system flow.

Mathematical application is possible by individually analyzing each facet of the system.

The system is so complex that reliance on the results of the two analytical approaches developed in this thesis could not be justified without comparing the results of the analytical approaches with actual results or at least simulated results.

The purpose of the two approaches performed in this study is to illustrate alternative approximation methods that can be performed without having to simulate a complex system. In the particular case of the previously described medium sized firm dealing in four types of products, the results of this study were approximately the same as those obtained in the simulation model. However, further study is necessary to validate the methods suggested in this thesis.

A. ANALYSIS OF ERROR FLOW

Each product as it flows through the system from the purchase of raw materials to sales of the finished product is assumed to have an equal probability of incurring the errors inherent in the accounting system.

For the purpose of this analysis product # 1 will be taken as representative of all products. According to the budgeted figures for product # 1 40,000 units of raw material will be purchased. The number

of units for each individual purchase of raw material is assumed to be a normally distributed random variable with a mean of 200 units and a standard deviation of 25 units.

Each lot of raw materials purchased has the same chance of quantity-counting errors and costing errors. Counting errors occur approximately 25% of the time and the error consists of a 10% understatement in the number of units purchased.

The costing errors occur approximately 10% of the time, and consists of an erroneous unit cost being applied to the units purchased. (Correct standard cost is \$ 13.5; erroneous standard cost is \$ 6.5)

The budgeted number of units requisitioned by production Dept. # 1 is 33.600 units.

Input		Raw Material Store		Output
40,000 units (μ=200 σ=25)	$\longrightarrow$	Raw Material Store	$\longrightarrow$	33,600 units (μ=150 σ=35)

The number of units per requisition is also assumed to be a normally distributed random variable with a mean of 150 units and a standard deviation of 35 units. When raw material lots are requisitioned, a 10% overstatement in the number of units actually requistioned occurs 15% of the time.

In addition to the 15% chance of an overstatement in the number of units requisitioned, there is a 10% chance that the cost accounting department will apply an erroneous unit cost.

Additionally there is an 8% chance that the cost accounting department will apply erroneous unit labor hours to the units requisitioned. (Correct standard is 0.06 hrs/unit; erroneous standard is 0.04 hrs/unit)

In the recording of the erroneous labor hours by the cost accounting department, there is a 5% chance that the type 1 standard labor rate ($ 6/hr) will be erroneously applied and a 5% chance that the type 2 standard labor rate ($ 5.6/hr) will be erroneously applied.

Additional errors occur in the application of labor hours. 8% of the time the cost accounting department will apply an erroneous standard overhead rate. (The correct value is $ 12.85/labor hr.; the erroneous value is $ 11.4/labor hr.).

Within Department #2 production units are overstated by 5%, 8% of the time. Additionally, the cost accounting department misapplies the standard labor hour.

Within Department # 2 the application of the correct and erroneous rate occur with the same probabilities as in Department # 1. (See error flowchart).

The budgeted number of finished goods transferred from producing Department # 2 to the finished goods storeroom is 30,280 units.

The number of units requisitioned by Department # 1 is a random variable (μ=150, σ=35) and as the products flow through the production process, additional quantity errors occur. Therefore, the reported number of units transferred to finished goods is also a random variable.

On the units transferred to finished goods, the cost accounting department misapplies the standard unit 8% of the time. (Correct: $ 16.929/unit, erroneous: $ 9.19/unit). On sales transactions no errors occur.

The above errors have been determined to be a potential threat to the system and would result in mistatements of the financial statements assuming zero compliance with the prescribed system of internal controls.

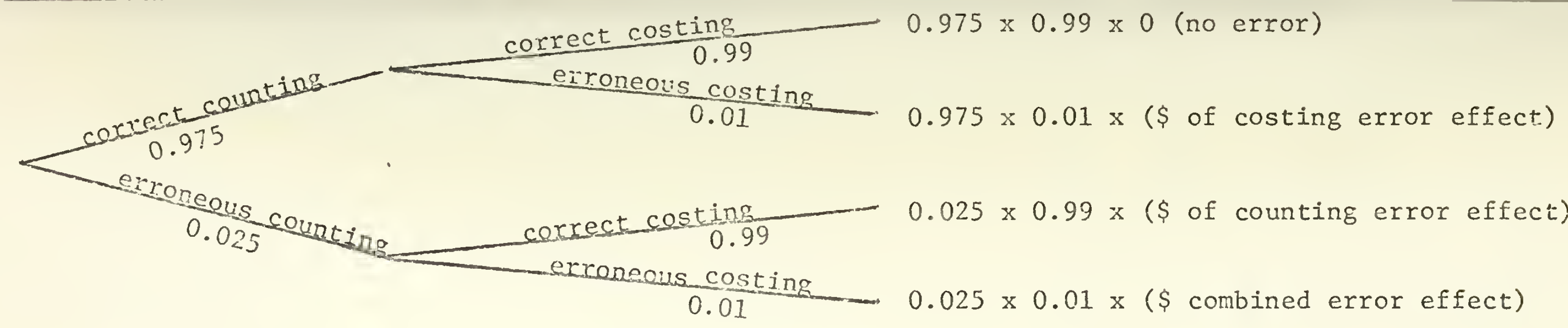
The auditor in the illustration is interested in assessing the total dollar amount of errors that might occur during the time period of reliance, escape the detection of prescribed controls and impact the financial statements. Of course these assessments would be performed on the basis of various assumptions related to internal control compliance. For example, if the auditor assumed that compliance with the prescribed internal control system might be 90% then a 25% chance of an undetected vendor's count error would be effectively reduced to a 2.5% chance. At this point the auditor should establish minimum compliance levels for the controls selected for reliance during his preliminary evaluation. The minimum (tolerable) levels should be set at values which the auditor considers necessary in the circumstances to justify his planned strategy of substantive procedures.

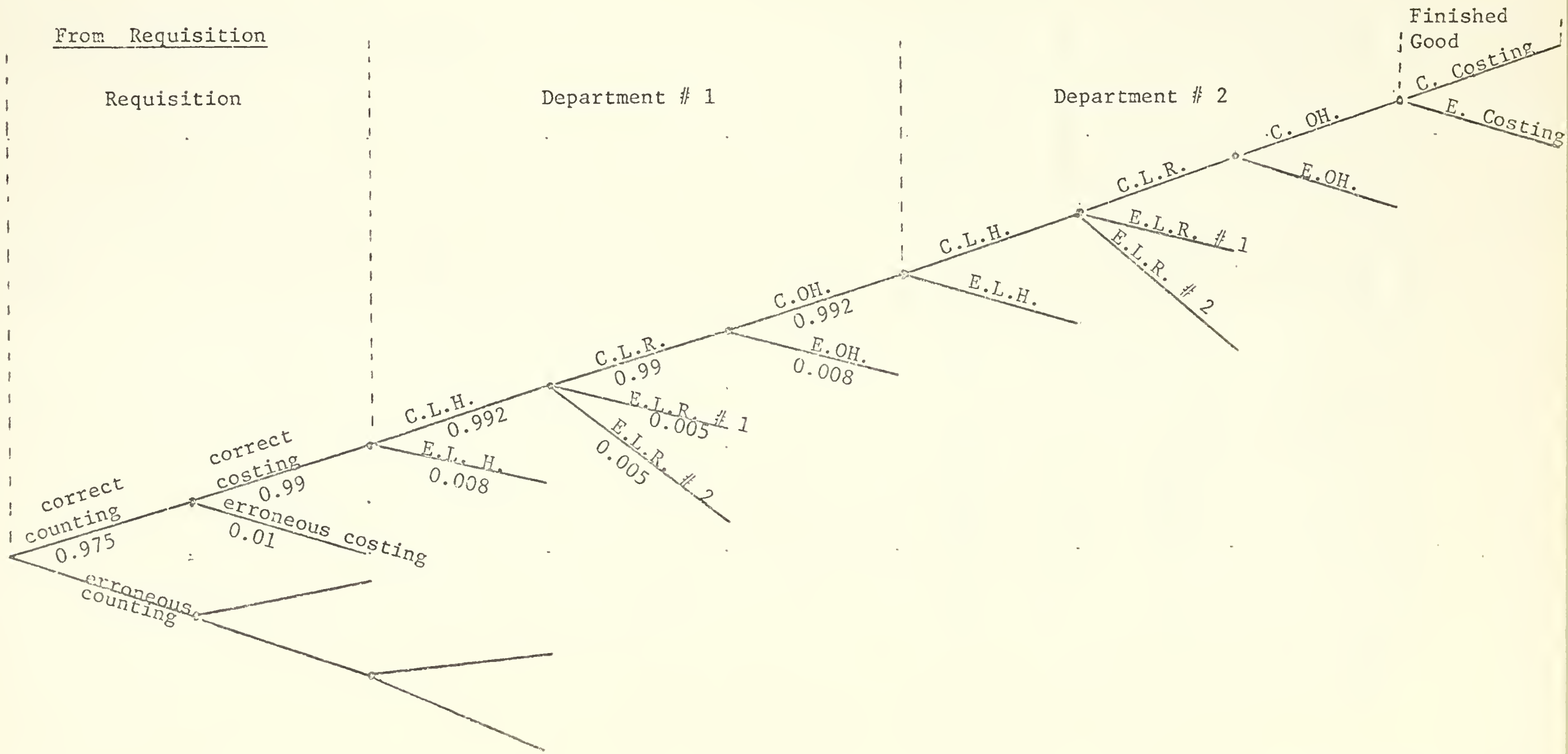
The financial statement consequences (Exhibit # 1) of the system's performance at various assumed levels of compliance with internal control furnish the auditor with an objective, rational basis for establishing minimum required reliability levels.

In developing a model, for convenience, a 90% level of effectiveness in the internal control system will be assumed. The fact that the internal control system is 90% effective in detecting errors is relevant to error frequency, meaning that every error frequency assumed above is reduced to one tenth of it's current level and the effectiveness has nothing to do with error magnitude.

One very useful method for interpreting and solving the problem described above, is the use of a tree diagram.

Each independent lot in each stage of the accounting flow is presented on the following tree diagram.


correct counting
0.975
erroneous counting
0.025
correct costing
0.99
erroneous costing
0.01
correct costing
0.99
erroneous costing
0.01
0.975 x 0.99 x 0 (no error)
0.975 x 0.01 x ($ of costing error effect)
0.025 x 0.99 x ($ of counting error effect)
0.025 x 0.01 x ($ combined error effect)


From Requisition
Requisition
Department # 1
Department # 2
Finished Good
correct counting
0.975
erroneous counting
correct costing
0.99
erroneous costing
0.01
C.L.H.
0.992
E.L.H.
0.008
C.L.R.
0.99
E.L.R. # 1
0.005
E.L.R. # 2
0.005
C.OH.
0.992
E.OH.
0.008
C.L.H.
E.L.H.
C.L.R.
E.L.R. # 1
E.L.R. # 2
C. OH.
E.OH.
C. Costing
E. Costing

Given a specific lot size (condition), the mean and variance of the error effect can be calculated directly from the above tree diagrams. For example at the purchase stage, given a particular lot size $X = X_i$, we have a computational formula.

E[input error] = (0.975)(0.99)(0; no error)+(0.975)(0.01)($ value of costing error effect)+(0.025)(0.99)($ value of counting error effect) +(0.025)(0.01)($ value of combined error effect).

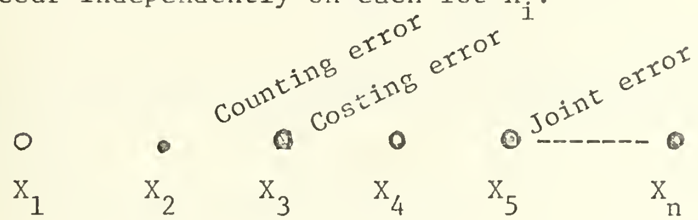
The above joint probabilities are computed using independence between counting errors and costing errors.

Also we compute variance in a similar way. The dollar value of each error effect will be analyzed later.

V. MEAN VALUE MODEL

A. ALGORITHM FOR MEAN VALUE OF THE ERRORS AT EACH PRODUCTION STAGE.

As seen in the previous chapter, the counting error and costing error can occur independently on each lot X_i.



As described earlier, each lot has

(1) a .025 chance of a counting error

(2) a .01 chance of a costing error

(3) a $(2.5)(1.0)(10^{-4})$ chance of a joint error and

(4) a 0.96525 chance of no errors.

Now, to demonstrate the nature and essence of this problem, and for simplification, let the random variable X_i (number of units in each lot) be degenerate at 200 units (mean value of raw material # 1), and then N (number of lots required to get 40,000 units) will be 200.

Then, among the 200 lots:

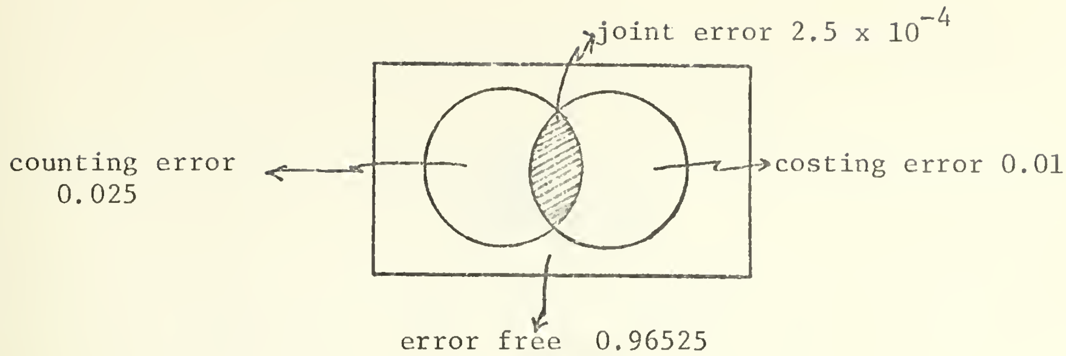
(1) 193.05 lots (200 lots x 0.96525) are expected to be free from errors.

(2) 5 lots (200 lots x 0.025) are expected to have a quantity error.

(3) 2 lots (200 lots x 0.01) are expected to have a counting error.

(4) 0.05 lots (200 lots x 0.00025) are expected to have joint error.

In all, errors are expected from 6.95 lots. (See Venn diagram below.)



Assuming lot sizes of 200 units the expected error effects are calcualted as follows:

(1) Quantity (Counting) error

If one lot has a counting error, the lot size will be understated by 20 units (200 units x 0.1) which results in a dollar error of \$ 270 (20 units x 13.5 unit cost). Therefore, the total average counting error during the period is expected to be \$270 x 5 lots = \$ 1,350. (understated).

(2) Costing error

If one lot has a costing error, the costing error will be (200 units)(Δ price)=(200)(6.5-13.5)=\$1,400. (understated).

Therefore, the total average costing error during the period is expected to be \$ 1,400 x 2 = \$ 2,800.

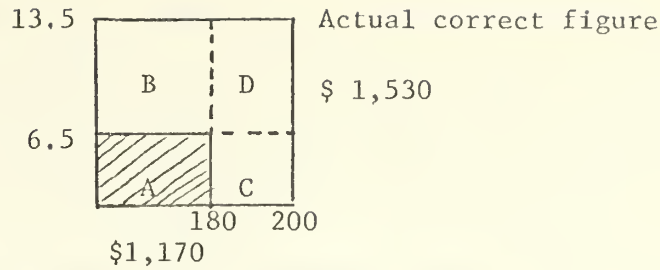
(3) The Joint error

The dollar value of 1 lot's joint error is

$$(180 \text{ units} \times \$\ 6.5) - (200 \text{ units} \times \$\ 13.5) = \$ -1{,}530$$

The total average joint error during the period is expected to be .05 x \$1,530 = \$ 76.5.

The joint error effect can be analyzed as follows:



A = (180)(6.5) = 1,170 (erroneous lot cost)

A+B+C+D = (200)(13.5) = 2,700 (correct lot cost)

B+C+D = 1,530 (joint error effect)

1 counting error = C+D = (20)(13.5) = $270

1 costing error = B+D = (7) (200) = 1,400.

Here we can see the joint error (B+C+D = 1,530) $\neq$ 1 counting error + 1 costing error (B+C+2D = 1,670).

We can compute joint error at each stage. But if this joint error effect is insignificant (immaterial) at each stage compared to the total combined error. We can save these computational efforts by substituting 1 counting error + 1 costing error (1,670) for the joint error (1,530). In this case the simplification causes a (140)(0.05) = $ 7 difference during the period.

The combined error effect during the period = $ -4,226.5 (-1,350 -2,800 -76.5).

The combined error effect by substitution during the period = $ -4,233.5 (-1,350 -2,800 -83.5).

The $ 7 difference is only 0.16% of the total error effect (7/4,226 = 0.00165). So for the tentative internal control evaluation purpose this approximation is tolerable. If it is significant we must compute the joint error exactly as (B+C+D). From the above reasoning, if we substitute one counting error + one costing error for one joint

error, then, the problem is much simpler than when we treat the joint error as a separate additional variable. With the simplification an average of 0.05 lots of the joint error among 200 lots is equivalent to 0.05 lots of costing error plus 0.05 lots of counting error.

This means we expect 5 lots with a counting error, 2 lots with a costing error and no lots containing joint error.

This situation holds true for W.I.P. inventory. For labor error, 1 lot's correct standard cost is (150 units)(6.2 \$/hr)(0.06 hr/unit) = \$ 55.8 (value added by Department # 1).

When a type 1 error is made at this stage, the standard cost is expected to be (165 units)(\$6.0/hr)(0.04 hr/unit) = \$39.6.

A type 2 error will be (165 units)(\$ 5.6/hr)(0.04 hr/unit) = \$ 36.96. These joint errors (counting error: 165 units instead of 150 units. Labor hour error: on product # 1 no error, labor rate error: \$ 6.0 for type 1 or \$ 5.6 for type 2 instead of correct \$ 6.2/hr) can be analyzed as follows:

(1) 1 lot's type 1 error

a. counting error ≈ (150 x 0.1)(0.06 hrs/unit)(612/hr) = + \$ 5.58.

b. labor hrs error ≈ (150)(0.04-0.06)(6.2/hr) = - 18.60

c. labor rate error≈(150) x 0.04 x (6.0-6.2) = - \$ 1.2.

The joint error (type 1), (39.6 - 55.8 = -16.2) is approximated to be equivalent to adding a. + b. + c. shown above (5.58 - 18.60 - 1.2 = - 14.22).

Similarly, 1 lot's type 2 joint error (36.96 - 55.8 = - 18.84) is approximated to be equivalent to adding the quantity error (+5.58), the labor hour error (-18.6) and the labor rate error (-5.4). The calculations above are also applicable to overhead errors.

1 lot's correct overhead standard cost = 150 units x 0.06 hrs/unit x \$ 12.85/hr = \$ 115.65.

1 lot's erroneous overhead standard cost = (150 x 1.1) x 0.04 x 11.4 = \$ 75.24.

This combined error (counting error, labor hr error and overhead rate error) can also be approximated as follows

1. Quantity error

   (150 x 0.1)(0.06)(12.85) = \$ 11.565

2. Labor hour error

   150 (0.04 - 0.06)(12.85) = \$ - 38.55

3. Overhead rate error

   (150)(-.06)(11.4 = 12.85) = \$ 13.05

Therefore, the dollar value of 1 lot's joint error is approximated to be equivalent to adding one quantity error + one labor hour error + one overhead rate error; meaning that the joint error is eliminated simply by computing each error independently.

These errors are independent so that when we compute the combined mean error of each stage we can add each mean error arithmetically. For example, when we compute the expected error of the ending inventory of each raw material (e.g. raw material # 1) we have expected error of ending inventory = expected input error - expected output error.

The total expected ending inventory error of raw materials is the sum of the expected errors of ending inventory for each raw material. This combining procedure is clarified in later sections.

B. ALGORITHM FOR VARIANCE

Because the probability of error for each lot is p = .025 the total number of error lots is the sum of independent Bernoulli trials. This

situation can be approximated by a Poisson distribution. Among 200 lots 5 lots (200 x 0.025) are the expected number of lots containing a counting error. The variance of the counting error is 5 (lots^2).

The mean and variance of the # of lots which have costing errors is 2 lots and 2 (lots^2) respectively. The standard deviation for the dollar value of the input counting errors for raw material # 1 is $\sqrt{5} \times 270 = \$\ 603.7$/year.

The standard deviation for the dollar value of input costing errors for raw material # 1 is $\sqrt{2} \times 1,400 = \$\ 1,980$/year.

Recall that these expressions are "quick and dirty" approximations to the actual case because of our assumption that all lots are of size 200. The objective of this section is simply to show the nature and accounting flow of the problem. Using the procedure described above, when we combine more than 2 independent erros for computational convenience we can utilize a probability theorem. When the relation $Y = ax$ holds. $E[Y] = a\,E[X]$ and $\text{Var}\,[Y] = a^2\ \text{Var}\,[X]$ are true.

The dollar value of input error $X = 270\ C_1 + 1,400\ C_2$ where C_1 and C_2 are number of lots having counting error and costing error respectively.

$$E[X] = 270\ E[C_1] + 1,400\ E[C_2] = 270 \times 5 + 1,400 \times 2 = 4,150$$

$$\text{Var}\,[X] = (270)^2\ \text{Var}\,(C_1) + (1,400)^2\ \text{Var}\,(C_2)$$

$$= (270)^2(5) + (1,400)^2(2)$$

Standard deviation of X, $\sigma_X = \sqrt{(270)^2(5)+(1,400)^2(2)}$

$$= \$\ 2,070 \tag{1}$$

Expression (1) is equivalent to multiplying the dollar value of one counting error (\$270) by the square root of (5 lots^2 of counting

error + 2 $(\frac{1,400}{270})^2$ lots^2 of costing error = 58.77 lots in terms of counting error) i.e., $\sigma_X = \sqrt{58.77} \cdot (270) = \$ 2,070$.

Because independence holds not only between the product, but also between each stage of accounting flow, we can compute means by summing each error according to the accounting flow. And we can compute variances or standard deviations simply by accumulating variances adjusted by the above procedures.

For the output of Raw Material # 1.

As calculated in the previous section, the expected number of counting error lots = $\frac{33,600}{150} \times 0.015 = 3.36$ lots, and the expected number of costing error lots = $\frac{33,600}{150} \times 0.01 = 2.24$ lots. One lot's counting error = (150 x 0.1) x 13.5 = \$ 202.5 and one lot's costing error = 150 x (6.5 - 13.5) = - \$1,050. The expected value of the combined output for Raw Material # 1 is 3.36 x 202.5 + (- 2.24 x 1,050) = - 1,672 (understated), the standard deviation is

$$\sqrt{3.36 \times (\frac{202.5}{270})^2 + 2.24 \times (\frac{1,050}{270})^2} \times 270 = 1,615.$$

Therefore the combined total error of ending inventory for Raw Material # 1 is obtained simply by subtraction. i.e., the combined total error for Raw Material # 1 = - 4,150 -(- 1,672) = - 2,478.

The corresponding combined standard deviation is

$$\sqrt{(2,070)^2 + (1,615)^2} = \# 2,625$$

or $$[5 + 2 \times (\frac{1,400}{270})^2 + 3.36 \times (\frac{202.5}{270})^2 + 2.24 \times (\frac{1,050}{270})^2]^{1/2}$$

$$\times 270 = \$ 2,625.$$

The above figures are summarized as follows:

Raw Material # 1

Counting error	Counting error
- 1,350	+ 680
Costing error	Costing error
- 2,800	- 2,352
Total Input error	Total Output error
- \$ 4,150	- \$ 1,672
Standard deviation	Standard deviation
\$ 2,070	\$ 1,615

Total ending inventory (input-output)

μ: - \$ 2,478

σ: \$ 2,625

The same algorithm is applicable for raw material # 2, raw material # 3 and raw material # 4. The totally combined figures of raw materials # 1, ... raw material # n follows the same procedure.

The results of the total ending raw material are shown below at each assumed internal control system reliability level. (95%, 90%,··· 75%).

A simple computer program for performing this computation is attached in Appendix B. This program is accompanied by a variable name dictionary. When we compare the figures generated by this program with the results of the computer simulation of Burn's (Exhibit II), we note little difference. (See Table III.)

However, validation of this procedure will be discussed in later sections of this paper. The computed figures for all products described in the problem are presented in Exhibit II.

TABLE III

COMPARISON OF THE TWO RESULTS

	MEAN		STANDARD DEVIATION	
RELIABILITY LEVEL	MEAN VALUE MODEL	SIMULATION	MEAN VALUE MODEL	SIMULATION
95%	- 3,166	- 3,150	3,843	3,800
90%	- 6,333	- 6,300	5,435	5,400
85%	- 9,499	- 9,400	6,656	6,300
80%	- 12,665	- 12,350	7,686	7,500
75%	- 15,832	- 15,750	8,593	8,300

EXHIBIT I

COMPUTER OUTPUT FOR RAW MATERIALS

```
$GO
                              RAW # 1
                             **********
95%
        QIN    -675.00        QOUT        340.20
        CIN   -1400.00        COUT      -1176.00
        TIN   -2075.00        TOUT       -835.80
      SIGIN    1463.64      SIGOUT       1141.79
                        TOTAL AVERAGE   -1239.20
                        TOTAL STNDVTN    1856.32

======================== P A G E   S K I P   S U P P R E S S E

                              RAW # 2
                             **********
        QIN    -709.75        QOUT        413.32
        CIN   -1479.00        COUT      -1435.50
        TIN   -2188.75        TOUT      -1022.17
      SIGIN    15[illegible]0.43      SIGOUT       1406.01
                        TOTAL AVERAGE   -1166.57
                        TOTAL STNDVTN    2122.81

                              RAW # 3
                             **********
        QIN    -276.25        QOUT        160.87
        CIN    1190.00        COUT       1155.00
        TIN     913.75        TOUT       1315.87
      SIGIN    1304.57      SIGOUT       1108.35
                        TOTAL AVERAGE    -402.12
                        TOTAL STNDVTN    1711.82
```

RAW # 4
****

QIN -320.00 QOUT 173.40

CIN 1392.00 COUT 1257.15

TIN 1072.00 TOUT 1430.55

SIGIN 1491.96 SIGOUT 1288.95
TOTAL AVERAGE -358.55

TOTAL STNDVTN 1971.63

TOTAL END RAW
****

MEAN -3166.45
STANDARD DEVIATION 3843.17

90%

RAW # 1
****

QIN -1350.00 QOUT 630.40

CIN -2800.00 COUT -2352.00

TIN -4150.00 TOUT -1671.60

SIGIN 2069.90 SIGOUT 1614.74
TOTAL AVERAGE -2478.40

TOTAL STNDVTN 2625.24

RAW # 2
****

QIN -1419.50 QOUT 826.65

CIN -2958.00 COUT -2871.00

TIN -4377.50 TOUT -2044.35

SIGIN 2249.20 SIGOUT 1388.40
TOTAL AVERAGE -2333.15

TOTAL STNDVTN 3002.11

RAW # 3
****

QIN -552.50 QOUT 321.75

CIN 2380.00 COUT 2310.00

TIN 1827.50 TOUT 2631.75

SIGIN 1344.94 SIGOUT 1567.44
TOTAL AVERAGE -804.25

TOTAL STNDVTN 2420.89

RAW # 4
****

QIN -640.00 QOUT 346.80

CIN 2784.00 COUT 2514.30

TIN 2144.00 TOUT 2861.10

SIGIN 2109.35 SIGOUT 1822.35
TOTAL AVERAGE -717.10

TOTAL STNDVTN 2788.31

TOTAL END RAW

****

MEAN -6332.89
STANDARD DEVIATION 5435.05

85%

RAW # 1
****

QIN -2325.00 QOUT 1020.60

CIN -4200.00 COUT -3528.00

TIN -6225.00 TOUT -2577.40

SIGIN 2535.10 SIGOUT 1977.64
TOTAL AVERAGE -3717.60

TOTAL STNDVTN 3215.25

RAW # 2
****

==================================== P A G E S K I P S U P P R E S S E

QIN -2129.25 QOUT 1239.97

CIN -4437.00 COUT -4306.50

TIN -6566.24 TOUT -3066.52

SIGIN 2754.70 SIGOUT 2435.28
TOTAL AVERAGE -3499.72

TOTAL STNDVTN 3676.31

RAW # 3
****

QIN -823.75 QOUT 482.63

CIN 3570.00 COUT 3465.00

TIN 2741.25 TOUT 3947.62

SIGIN 2259.59 SIGOUT 1919.71
TOTAL AVERAGE -1206.38

TOTAL STNDVTN 2964.97

RAW # 4
****

QIN -960.00 QOUT 520.20

CIN 4176.00 COUT 3771.45

TIN 3216.00 TOUT 4291.64

SIGIN 2584.15 SIGOUT 2232.52
TOTAL AVERAGE -1675.65

TOTAL STNDVTN 3414.97

TOTAL END RAW

****

MEAN -9499.34
STANDARD DEVIATION 6656.55

60%

RAW # 1
****

QIN -2700.00 QOUT 1360.00

CIN -5600.00 COUT -4704.00

TIN -8300.00 TOUT -3343.20

SIGIN 2327.28 SIGOUT 2283.50

TOTAL AVERAGE -4956.00

TOTAL STDDVTN 3712.65

RAW # 2
****

QIN -2839.00 QOUT 1653.30

CIN -5915.99 COUT -5742.00

TIN -8751.99 TOUT -4083.70

SIGIN 3180.85 SIGOUT 2812.03

TOTAL AVERAGE -4665.29

TOTAL STDDVTN 4245.62

RAW # 3
****

QIN -1105.00 QOUT 643.50

CIN 4760.00 COUT 4620.00

TIN 3655.00 TOUT 5263.50

SIGIN 2509.15 SIGOUT 2216.00

TOTAL AVERAGE -1633.50

TOTAL STDDVTN 3423.65

RAW # 4
****

QIN -1280.00 QOUT 693.60

CIN 5567.79 COUT 5028.59

TIN 4287.99 TOUT 5722.19

SIGIN 2983.42 SIGOUT 2577.90

TOTAL AVERAGE -1434.20

TOTAL STDDVTN 3943.26

TOTAL END RAW

****

MEAN -12666.74

STANDARD DEVIATION 7636.33

75%

RAW # 1
****

QIN -3375.00 QOUT 1701.00

CIN -7000.00 COUT -5880.00

TIN -10375.00 TOUT -4179.00

SIGIN 2272.00 SIGOUT 2553.12

TOTAL AVERAGE -4136.00

TOTAL STDDVTN 4150.96

RAW # 2
****

QIN -3543.75 QOUT 2066.63

CIN -7394.99 COUT -7177.49

TIN -10943.74 TOUT -5110.87

SIGIN 3556.30 SIGOUT 3143.93
TOTAL AVERAGE -5832.37

TOTAL STNDVTN 4746.74

RAW # 3
****

QIN -1381.25 QOUT 804.38

CIN 5950.00 COUT 5775.00

TIN 4563.75 TOUT 6579.37

SIGIN 2917.11 SIGOUT 2473.34
TOTAL AVERAGE -2010.63

TOTAL STNDVTN 3827.76

RAW # 4
****

QIN -1600.00 QOUT 867.00

CIN 6359.99 COUT 6285.75

TIN 5359.99 TOUT 7152.75

SIGIN 3326.13 SIGOUT 2882.17
TOTAL AVERAGE -1792.75

TOTAL STNDVTN 4408.70
TOTAL END RAW
****

MEAN -15832.25
STANDARD DEVIATION 8593.58

* FINAL VALUE *

	MEAN	STNDEVIATION
95%	-3166.45	3843.17
90%	-6332.89	5435.05
85%	-9499.34	6656.55
80%	-12665.79	7686.33
75%	-15832.25	8593.53

SAGE OBJECT CODE= 3192 BYTES,ARRAY AREA= 60 BYTES,TOTAL AREA

E TIME= 0.22 SEC,EXECUTION TIME= 0.43 SEC, WATFIV - VERSION 1 LEV

EXHIBIT II

SAMPLE OF OUTPUT DATA GENERATED

BY THE RAW MATERIALS COMPUTER PROGRAM

(Simulation)

	Run 1	Run 2	Run 3	Run 4	Run 5
Level of compliance with pertinent controls during the run	95%	90%	85%	80%	75%
Mean dollar value of net error which might result in Dec. 31 raw material account balance.*	($3,150)	($6,300)	($9,400)	($12,350)	($15,750)
Mean dollar error as a per cent of budgeted Dec. 31 raw materials account balance (i.e. $ 473,500).	.7%	1.3%	2.0%	2.6%	3.3%
Standard deviation of expected dollar error.	$3,800	$5,400	$6,300	$7,500	$8,300

* (   ) indicates understatement of raw materials account balance.

EXHIBIT III

ERROR EFFECTS OF TOTAL INVENTORIES

****

ABILITY 95.%

UCT#		RAW	LABOR	OH	WIP	FIN
1	AVE	-1239.20	-14.12	57.21	-342.34	-450.37
	SIG	1856.32	20.25	93.47	1584.80	1094.88
2	AVE	-1166.57	-13.41	152.33	-372.45	-510.80
	SIG	2122.81	31.11	156.74	1949.81	1341.40
3	AVE	-402.12	17.12	176.20	327.45	1181.74
	SIG	1711.82	19.08	73.05	1528.48	1049.81
4	AVE	-358.55	15.42	201.30	579.72	1067.55
	SIG	1971.63	28.16	126.80	1688.71	1083.27
		TOTAL ERROR				
	AVE	-3166.45	5.01	587.04	192.37	1288.13
	SIG	3843.17	50.35	233.92	3391.36	2296.46

****

ABILITY 90.%

UCT#		RAW	LABOR	OH	WIP	FIN
1	AVE	-2478.40	-28.24	114.42	-684.68	-900.74
	SIG	2625.24	28.64	132.18	2241.25	1548.39
2	AVE	-2333.15	-26.82	304.65	-744.90	-1021.61
	SIG	3002.11	44.00	221.66	2757.45	1897.03
3	AVE	-804.25	34.24	352.40	654.90	2363.49
	SIG	2420.89	26.99	103.31	2161.59	1484.66
4	AVE	-717.10	30.83	402.61	1159.43	2135.10
	SIG	2788.31	39.82	179.32	2388.20	1531.98
		TOTAL ERROR				
	AVE	-6332.89	10.02	1174.08	384.75	2576.25
	SIG	5435.05	71.20	330.81	4796.11	3247.68

****

RELIABILITY 85.%

PRODUCT#		RAW	LABOR	OH	WIP	FIN
1	AVE	-3717.59	-42.36	171.63	-1027.02	-1351.10
	SIG	3215.25	35.07	161.89	2744.96	1896.39
2	AVE	-3499.72	-40.22	456.98	-1117.36	-1532.41
	SIG	3676.81	53.89	271.48	3377.17	2323.38
3	AVE	-1206.37	51.36	528.60	982.35	3545.23
	SIG	2964.97	33.06	126.53	2647.40	1818.33
4	AVE	-1075.65	46.25	603.91	1739.15	3202.66
	SIG	3414.97	48.77	219.62	2924.93	1876.29
			TOTAL ERROR			
	AVE	-9499.33	15.03	1761.12	577.12	3864.38
	SIG	6656.55	87.21	405.16	5874.01	3977.58

****

RELIABILITY 80.%

PRODUCT#		RAW	LABOR	OH	WIP	FIN
1	AVE	-4956.79	-56.47	228.84	-1369.36	-1801.47
	SIG	3712.65	40.50	186.94	3169.60	2189.76
2	AVE	-4666.29	-53.63	609.31	-1489.81	-2043.21
	SIG	4245.62	62.23	313.48	3899.62	2682.81
3	AVE	-1608.50	68.48	704.80	1309.79	4726.98
	SIG	3423.65	38.17	146.11	3056.95	2099.63
4	AVE	-1434.20	61.66	805.22	2318.86	4270.21
	SIG	3943.26	56.31	253.60	3377.42	2166.55
			TOTAL ERROR			
	AVE	-12665.77	20.04	2348.16	769.48	5152.50
	SIG	7686.33	100.70	467.84	6782.72	4592.91

****

RELIABILITY 75.%

PRODUCT#		RAW	LABOR	OH	WIP	FIN
1	AVE	-6196.00	-70.59	286.05	-1711.70	-2251.84
	SIG	4150.86	45.28	209.00	3543.73	2448.23
2	AVE	-5832.87	-67.04	761.64	-1862.26	-2554.01
	SIG	4746.74	69.57	350.48	4359.91	2999.47
3	AVE	-2010.62	85.60	881.00	1637.25	5908.72
	SIG	3827.76	42.67	163.35	3417.78	2347.45
4	AVE	-1792.75	77.08	1006.52	2898.58	5337.76
	SIG	4408.70	62.96	283.53	3776.07	2422.27

TOTAL ERROR

AVE	-15832.24	25.05	2935.20	961.86	6440.63
SIG	8593.58	112.58	523.06	7583.31	5135.03

CORE USAGE OBJECT CODE= 7752 BYTES,ARRAY AREA= 60 BYTES,TOT

COMPILE TIME= 0.67 SEC,EXECUTION TIME= 0.68 SEC, WATFIV - VERSIO

VI. STOCHASTIC MODEL

In the previous chapter an approximate analytical model of the internal control process was investigated. In this chapter, the illustrative control subsystem is treated as a stochastic process in which errors of the various types are considered to be generated at random in accordance with probability laws having known means and variances. Expressions for the expected dollar value of errors and variances are derived, and the results are compared with the simulated results and the results from the previous chapter.

Let X_i be the number of units of raw material in the i^{th} lot delivered and let N be the number of lots required to obtain the 40,000 units needed for the three month period from September through the end of December. Because the lot sizes vary, the X_i's and N are random variables. The X_i's are assumed to be independent non-negative random variables each having the same (unspecified) probability distribution with mean $\mu_i = 200$ and standard deviation $\sigma_i = 25$. With these assumptions the X_i's form a renewal process[5] (the lot size is thought of as analagous to the interarrival times in the usual application of renewal theory). Define the random variable S_n as follows:

$$S_o = 0$$

$$S_n = \sum_{i=1}^{n} X_i \, , \; i = 1,2, \ldots .$$

---

[5] Sheldon M. Ross "APPLIED PROBABILITY MODELS WITH OPTIMIZATION APPLICATIONS (Holden Day, 1970).

Then, it is easily seen that $N = \max \{n: S_n \leq 40{,}000\}$. In the renewal theory terminology, N corresponds to the number of renewals in the "interval" [0, 40,000]. Each time a lot arrives the lot will have a counting error, a costing error, a joint error or no error, the outcome being random with the probabilities given earlier.

The dollar value of the errors is the figure of interest. Let e_i be the dollar value of the error for the i^{th} lot, then

$$e_i = \begin{cases} -(0.1X_i)(13.5) & \text{with probability } (0.025)(0.99) = 0.02475 \\ -(13.5-6.5)X_i & \text{with probability } (0.01)(0.975) = 0.00975 \\ -1.35X_i - 7(.9X_i) & \text{with probability } (0.01)(0.025) = 0.00025 \\ 0 & \text{with probability } (1-0.0345 + 0.00025) \\ & = 0.96575 \end{cases}$$

$$E[e_i/X_i=x] = -1.35x(0.02475) - 7x(0.00975) - 7.65x(0.00025)$$
$$= -(0.0334125 + 0.06825 + 0.0019125)x = -0.103575x$$

therefore,

$$E[e_i] = E[E[e_i/X_i]] = E[-0.103575x] = -20.715$$

Similarly

$$E[e_i^2/X_i=x] = 1.8225x^2\ (0.02475) + 49x^2\ (0.00975) + 58.5225x^2\ (0.00025)$$
$$= x^2\ (0.5374874)$$

so that $\text{var}\ [e_i/X_i = x] = 0.5374874x^2 - 0.0107277x^2 = 0.5267597x^2$

and $\text{var}\ [e_i] = E[\text{var}(e_i/X_i)] + \text{Var}\ (E[e_i/X_i])$

$$= E[0.5267597X_i^2] + \text{Var}\ (-0.103575X_i)$$

$$= 0.5267597\ E[X_i^2] + 0.0107277\ \text{var}\ (X_i)$$

$$= 0.5267597\ [\text{var}(X_i) + E^2[X_i]] + 0.0107277\ \text{var}\ (X_i)$$

$$= 0.5374874\ \text{var}\ (X_i) + 0.5267597\ E^2[X_i]$$

$$= 335.92962 + 21{,}070.388 = 21{,}406.317$$

and the standard deviation of the dollar value of the i^{th} lot error is 146.309.

The total dollar value of raw material error overall lots is given by $e = \sum_{i=1}^{N} e_i$, and from renewal theory[6], the distribution of N is approximately normal with mean $\frac{t}{\mu} = \frac{40{,}000}{200} = 200$ and variance $\frac{\sigma^2 \cdot t}{\mu^3}$ $= 3.125$ where $t = 40{,}000$.

Therefore $E[e] = E[E[e/N=n]] = -\ 20.715\ E[N] = -\ 4{,}143$

$$\text{Var}\ (e) = \text{Var}\ (E[e/N]) + E[\text{var}(e/N)]$$

$$E[e/N] = -\ 20.715N$$

$$\text{Var}(E[e/N]) = 429.11122\ \text{Var}(N) = 1{,}340.9725$$

$$\text{Var}[e/N=n] = \text{Var}(\sum_{i=1}^{n} e_i) = n\ \text{Var}\ (e_i) = n \cdot (21{,}406.317)$$

$$E[\text{Var}(e/N)] = E[N](21{,}406.317) = 4{,}281{,}263.4$$

$$\text{Var}(e) = 4{,}282{,}604.2$$

$$SD(e) = 2{,}069.4454 \approx 2{,}069$$

As above we have input error of raw material # 1 with mean dollar value - \$4,150 and standard deviation \$ 2,069. Similarly we compute the <u>output error</u> of raw material # 1 as follows:

[6] D. R. Cox "RENEWAL THEORY"

here $t = 33{,}600$, $E[X_i] = 150$, and $\sigma X_i = 35$.

$$e_i = \begin{cases} +\ (0.1X_i)(13.5) & \text{with probability } (0.015)(0.99) \\ -\ (7X_i) & \text{with probability } (0.01)(0.985) \\ +\ (1.35X_i) - 7(1.1X_i) = -\ 6.35X_i & \text{with probability } (0.01)(0.015) \\ 0 & \text{otherwise} \end{cases}$$

$$E[e_i/X_i = x] = 1.35x\ (0.01485) - 7x\ (0.00985) - 6.35x\ (0.00015)$$

$$= -\ 0.049855x.$$

$$E[e_i] = -\ 0.049855\ E[X_i] = -\ 7.47825$$

$$E[e_i^2/X_i = x] = 0.0270641x^2 + 0.48265x^2 + 0.0060483x^2 = 0.5157624x^2$$

$$\text{Var}[e_i/X_i = x] = 0.5157624x^2 - 0.0024855x^2 = 0.5132769x^2$$

$$\text{Var}(e_i) = E[\text{Var}(e_i/X_i)] + \text{Var}(E[e_i/X_i])$$

$$= E[0.5132769X_i^2] + \text{Var}\ (-0.049855X_i)$$

$$= 0.5132769\ E[X_i^2] + 0.0024855\ \text{Var}(X_i)$$

$$= 0.5132769\ [\text{Var}(X_i) + E^2[X_i]] + 0.0107277\ \text{Var}(X_i)$$

$$= 0.5240046\ \text{Var}(X_i) + 0.5132769\ E^2(X_i)$$

$$= 12{,}190.635$$

$SD(e_i) = 110.4112$

$E[N] = \frac{33,600}{150} = 224$

$Var(N) = \frac{\sigma^2 \cdot t}{\mu^3} = \frac{(1225)(33,600)}{(150)^3} = 12.195545$

$E[e] = -\ 7.47825\ \ E[N] = -\ 1,675.128$

$Var(e) = Var(E[e/N]) + E[Var(e/N)]$

$E[e/N] = -\ 7.47825N$

$Var(E[e/N]) = 55.924223\ Var(N) = 682.02637$

$Var(e/N = n) = n\ Var(e_i) = 12,190.635n$

$E[Var(E/N)] = 2,730,702.2$

$Var(e) = 2,731,384.2$

$SD(e) = 1,652.6899 \approx 1,653$

Therefore, we have the output error of raw material # 1, with mean dollar value - $ 1,675 and standard deviation $ 1,653. The error effects on ending raw material # 1 are calculated by combining input error and output error. i.e., the mean value of error effect on ending raw material # 1 is the mean value of input error minus the mean value of output error = -4,150 + 1,675 = - 2,475

The standard deviation of this error effect is $(4,282,604.2 + 2,731,384.2)^{1/2} = 2,648.39 \approx 2,648$.

A summary of errors of raw material # 1 is tabulated below.

Errors of Raw Material # 1

$\mu_i = -4{,}150$	$\mu_o = -1{,}675$
$\sigma_i = 2{,}069$	$\sigma_o = 1{,}653$

$\mu = -2{,}475$

$\sigma = 2{,}648$

In this way for the errors of raw material # 2.

Errors of Raw Material # 2

$\mu_i = -4{,}370$	$\mu_o = -2{,}049$
$\sigma_i = 2{,}262$	$\sigma_o = 2{,}034$

$\mu = -2{,}321$

$\sigma = 3{,}042$

Where $E(X_i) = 180$ $t_i = 34{,}000$ $SD(X_i) = 30$ for the input

$E(X_i) = 150$ $t_o = 33{,}000$ $SD(X_i) = 35$ for the output.

correct price = 16.7 erroneous price = 8.

Errors of Raw Material # 3

$\mu_i = +1{,}821$	$\mu_o = +2{,}635$
$\sigma_i = 1{,}846$	$\sigma_o = 1{,}605$

$\mu = -814$

$\sigma = 2{,}446$

Errors of Raw Material # 4

μ_i = 2,137	μ_o = 2,865
σ_i = 2,140	σ_o = 1,867

$$\mu = -728$$

$$\sigma = 2,840$$

The total comined error of raw material # 1, # 2, # 3, and # 4 at 90% compliance level

$$\mu_T = -6,338$$

$$\sigma_T = 5,505$$

At 90% compliance level we have the results,

	COMPUTER SIMULATION	MEAN VALUE METHOD	STOCHASTIC MODEL
μ_T	- 6,300	- 6,330	- 6,338
σ_T	5,400	5,436	5,505

The combined errors of the total inventories are computed by the Mean Value Method in Exhibit III.

VII. SUMMARY AND CONCLUSION

In Chapter V we computed the counting error of the i^{th} lot. Using Mean Value Method to be $e_i = -(0.025)(1.35)X_i + 7X_i\ (0.01) = -0.10375X_i$ as an approximation.

In Chapter VI we computed

$$e_i = -\ (0.025)(0.99)(1.35)X_i + (0.01)(0.975)\ 7X_i + (7.65)(0.025)(0.01)$$

$= -\ 0.103575X_i$ as an exact way. Also, in Chapter V, we assumed that N was deterministic (not a random variable). That approximation is justifiable only when Var(N) is small enough that the random variable N is concentrated around it's mean value.

In this particular problem, there is not much difference between the results of the two methods presented, but in using the Mean Value Method we should check to see if it is reasonable to treat N as deterministic and to approximate the joint error as we do. If the approximations are not reasonable, the Exact Method should be used.

The advantages of using the Mean Value Method (when it is appropriate) are significant.

(1) The method should be easily understood by an auditor who does not possess an extensive mathematical background.

(2) In the accounting flow, especially in W.I.P. inventories, the accounting flows are so complex per se, that using the exact Stochastic Method could be very difficult and thus might be volnerable to computing errors.

(3) The Mean Value Method is a good "quick and dirty" Method that can serve the preliminary evaluation of internal control systems. It is

easily and practically usable by auditors who do not have extensive backgrounds in Computer Simulation and Probability Theory.

The user must realize that use of the Mean Value Method as an approximation is valid only when the conditions mentioned earlier are satisfied.

In this problem we initially assumed that the lot size X_i is normally distributed, but in using the Stochastic Method the distribution was not necessary; all that was required was the mean and variance of X_i.

The information needed to use the Stochastic Method is summarized below:

(1) Error Magnitude and Error Frequency

(2) μ_{X_i} and σ_{X_i} (we do not need to know the distribution of X_i)

(3) $E[N] = \frac{t}{\mu_x}$, $\text{Var}\,(N) \approx \frac{\sigma_x^2\, t}{\mu_x^3}$

Where t is the budgeted number of units during the period, and N is approximately normal distribution.

(These expressions for E[N] and Var[N] are approximations which will be very good provided t/μ is large.)

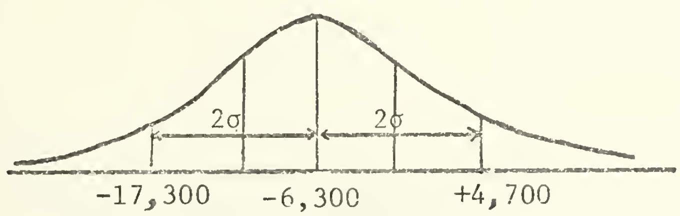
The statistical results computed in this chapter (at the 90% compliance level) offer objective bases for evaluating the raw materials subsystems as a whole, and the statistical results at variable feasible compliance levels furnish an objective, rational basis for establishing tolerable compliance levels.

The mean value of the total errors - 6,338 calculated in Chapter VI show that 90% compliance with pertinent controls might assure accuracy in the December 31st raw materials account to the degree of a \$ 6,338 understatement. This \$ 6,338 figure is a mean error statistic; hence,

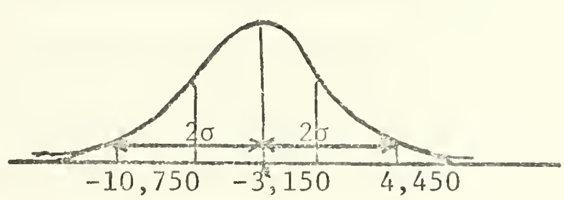
the auditor could gain further insight concerning the potential variability of the raw materials error at 90% compliance by using the standard deviation of $ 5,500.

By the Central Limit Theorem the total combined error figure is approximately normal distributed with the mean - 6,300, standard deviation 5,500. Therefore the auditor has 95.44% of confidence that the raw materials error would be somewhere between the precision limits of a $ 4,700 overstatement of the account and a $17,300 understatement of the account.

At the 90% Compliance level



and at a 95% Compliance level (computer simulation results)



the auditor has 95% confidence intervals of - 3,150 $\pm$ 2σ = - $ 10,750 and $ 4,450.

In this way, the auditor can decide the tolerable compliance level in his audit strategy and this procedure would be used in sampling to test pertinent raw materials controls.

APPENDIX A

SUMMARY DESCRIPTION OF BURNS' ILLUSTRATIVE PROBLEM

DESCRIPTION OF THE OPERATIONS AND THE INVENTORY ACCOUNTING AND CONTROL SYSTEM OF THE HYPOTHETICAL FIRM

DESCRIPTION OF THE OPERATIONS OF THE HYPOTHETICAL FIRM

The firm is assumed to be engaged in the machining and sales of alloy-cast-iron pipe fittings. The firm produces a full line of over two thousand pipe fittings in various sizes and designs. The inventory model and audit tests performed in Chapter III will be concerned with four products from the firm's total line. The four pipe fittings are assumed to constitute one distinct product line of the firm. This line is sold, in large lots primarily to building contractors, the petroleum and chemical industry, and industrial equipment manufacturers. The four products selected are assumed to constitute

a material percentage of total annual sales volume. They are further assumed to be representative of the total product line of the firm for purposes of testing the inventory accounting system. A detailed cost build up of the four products is presented as Exhibit II-1.

DESCRIPTION OF THE RAW MATERIALS ACCOUNTING AND CONTROL SYSTEM

Raw Materials Policies

All raw materials are carried on the company's records at standard cost. At year end, the raw materials account is adjusted to the lower of standard cost or market.

A perpetual inventory of raw material quantities on hand and on order is maintained by the production control department. This record is updated daily for raw material orders requisitioned, orders received from vendors and materials requisitioned into production.

A physical inventory is taken each year on September 30. The perpetual and financial records are adjusted to the physical inventory. The company relies upon financial records to provide an accurate account of inventory transactions between September 30 and the December 31 year end.

Raw Materials Accounting and Control Procedures

Production control clerks review the raw material perpetual stock control records and production schedule each day. They prepare purchase order requisitions for raw materials. The size of an individual order depends upon many factors but rarely exceeds three hundred units. Company policy is to maintain approximately a two month's supply of raw material on hand at all times. This policy safeguards against raw material shortage and supply problems.

EXHIBIT II-1

UNIT PRODUCT COST BUILDUP

	Product Number 1	Product Number 2	Product Number 3	Product Number 4
Direct Material				
Type of material	R.M. 1	R.M. 2	R.M. 3	R.M. 4
Units required	1	1	1	1
Spoilage, scrap, shrinkage	0	0	0	0
Standard cost of material	$13.5000	$16.7000	$ 6.5000	$ 8.000
Total Direct Material/Unit	$13.5000	$16.7000	$ 6.5000	$ 8.000
Direct Labor				
Department I				
Standard dir. lbr. hrs./unit	.06 hr.	.09 hr.	.04 hr.	.06 hr.
Standard dir. lbr. rate	$6.20/hr.	$6.20/hr.	$6.20/hr.	$6.20/hr.
Total standard dir. lbr. charge	$ .3720	$ .5580	$ .2480	$ .3720
Department II				
Standard dir. lbr. hrs./unit	.04 hr.	.06 hr.	.04 hr.	.07 hr.
Standard dir. lbr. rate	$5.60/hr.	$5.60/hr.	$5.60/hr.	$5.60/hr.
Total standard dir. lbr. charge	$ .2240	$ .3360	$ .2240	$ .3920
Total Direct Labor/Unit	$ .5960	$ .8940	$ .4720	$ .7640
Burden				
Department I				
Standard dir. lbr. hrs/unit	.06 hr.	.09 hr.	.04 hr.	.06 hr.
Standard burden rate	$12.85/hr.	$12.85/hr.	$11.40/hr.	$11.40/hr.
Total standard burden charge	$ .7710	$ 1.1565	$ .4560	$ .6840
Department II				
Standard dir. lbr. hrs/unit	.04 hr.	.06 hr.	.04 hr.	.07 hr.
Standard burden rate	$51.55/hr.	$51.55/hr.	$44.05/hr.	$44.05/hr.
Total standard burden charge	$ 2.0620	$ 3.0930	$ 1.7620	$ 3.0835
Total Burden/Unit	$ 2.8330	$ 4.2495	$ 2.2180	$ 3.7675
Total Unit Standard Cost	$ 16.9290	$ 21.8435	$ 9.1900	$ 12.5315

Purchase order requisitions are prepared on a standardized two-copy form. The original copy is forwarded to the purchasing department and the second copy is filed in production control in a temporary-hold file. (See Figure II-1)

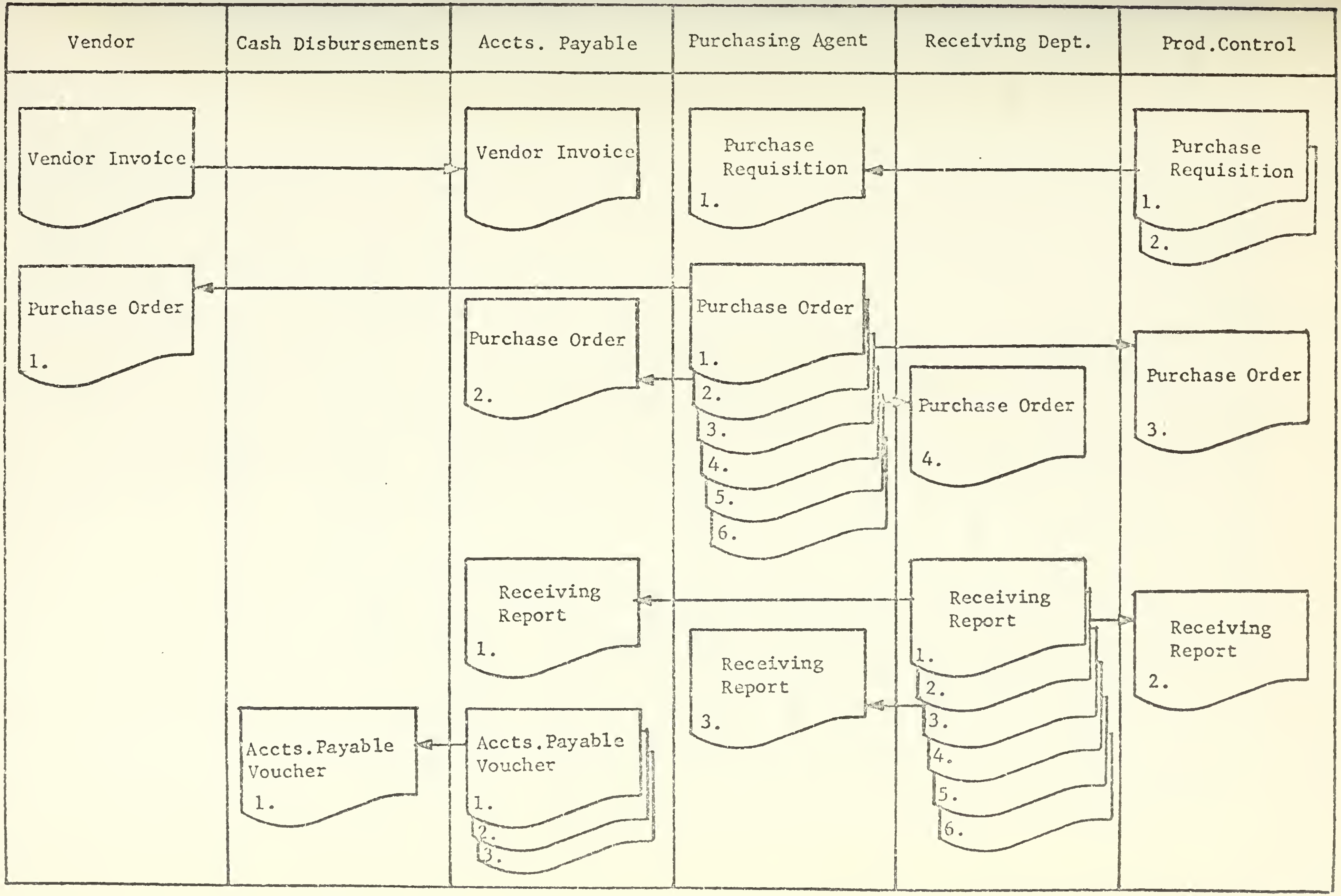
The purchasing agents prepare the purchase orders from the purchase requisitions. A separate purchase order is prepared for each raw material order. The agent obtains standard material prices from the standard cost file maintained in the cost accounting department. The purchasing agent extends the purchase order and forwards it to a clerk who distributes the individual copies of the form and prepares the vendor copy for mailing. (See Figure II-1)

The purchase order is a six-copy form. The original copy is the vendor copy. The second copy is the accounts payable copy. The third copy is forwarded to production control where it serves as an open order file and is matched with the purchase requisition to confirm the order. The fourth copy of the purchase order is the receiving department copy. This copy is maintained by the receiving department and compared with the shipment when the order arrives. The fifth and sixth copies of the purchase order are filed in the purchasing department. The fifth copy is filed alphabetically by vendor. The sixth copy is filed in numerical sequence.

The orders shipped by the vendor foundries often vary slightly from the quantities stated on the purchase order. Vendor foundries produce the alloy castings especially for the company in accordance with rigid specifications. Vendors often experience considerable variation in production yields and have thus obtained permission from the company to ship all good castings produced. If quantities produced

DOCUMENT FLOW

RAW MATERIAL PURCHASING AND RECEIVING



by the foundries vary by a material amount from quantities ordered by the company, however, the vendor foundry is expected to contact the company for special permission to ship. If shipment is permitted, the purchasing agent contacts the receiving department and the receiving copy of the purchase order is changed to reflect the special circumstances.

Receiving department personnel inspect and weigh-count all incoming raw material shipments. All local vendors make delivery via their own trucks. The exact quantity received for each shipment is determined by the receiving department personnel and the vendor company's truck driver at the time of delivery. The procedure has been agreed upon and followed by the company and its vendors for many years. All orders are shipped to the company in large wooden boxes supplied by the vendor. These boxes accompany the castings throughout the manufacturing process up to the point of transfers of completed castings to finished goods inventory. After castings are transferred to finished goods the boxes are returned to raw materials stores and are then picked up by the vendors' truck drivers from time to time.

Receiving department personnel prepare a six-copy, pre-numbered, receiving form for each order received. (See Figure II-1) Receiving forms are prepared chronologically in numberical sequence. The following data are entered upon the six-copy receiving form:

1. Name of vendor
2. Purchase order reference number
3. Quantity ordered
4. Quantity received
5. Description of item
6. Part number of item
7. Date

8. Signature of checker

9. Signature of truck driver

10. Date signed by truck driver

The original number one copy of the receiving form is the accounts payable copy. The second copy is forwarded to production control for posting to the perpetual stock control records. The third copy of the receiving form is forwarded to the purchasing agent to confirm receipt of the order. The fourth copy of the receiving form is placed in a clear-plastic envelope and attached to the box containing the order. The fifth and sixth copy of the receiving form are filed in the receiving department. The fifth copy is filed alphabetically by vendor. The sixth copy is filed in numerical sequence.

After receiving and inspection are completed the raw material orders are moved to the raw materials storage area where they are stored until use. Orders are stored in their wooden shipping boxes in the raw materials storage area.

Accounts payable clerks match vendors' invoices with receiving reports, vendors' shippers, and the purchase order and voucher the invoice for payment. Standard costs noted on the purchase order are used in the vouchering operation. Inventory is charged at standard cost extended by the quantity actually received. Purchase price variances are recorded in an appropriate price variance account. Problems arising from a disagreement between the quantity of material received and quantity billed are charged or credited to detailed unsettled claims accounts at actual price. Claims are cleared by the purchasing department.

Production orders are issued to the foreman of the department

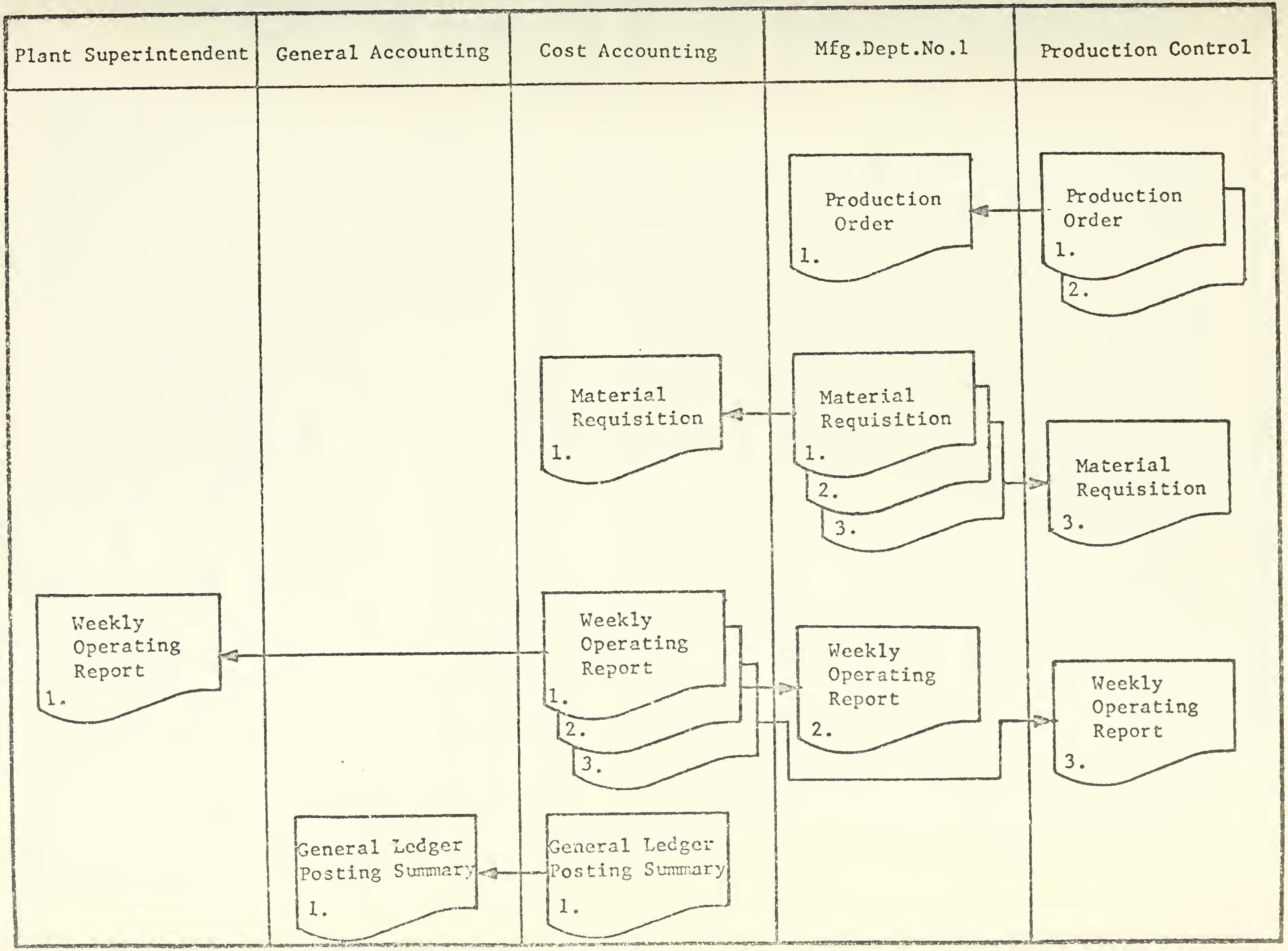
performing the first operation on the product. Orders are scheduled by production control one month in advance and released to foreman a week before production is scheduled to begin. Individual production orders vary from approximately 50 units to 250 units in size but average about one hundred fifty units. Orders larger than two hundred fifty units are rarely scheduled as they produce material handling problems on the shop floor.

The company's production order is a pre-numbered, two-copy form (See Figure II-2) All copies of the form provide spaces for the following information:

1. Product part number
2. Product description
3. Suggested number to be produced
4. Date production is to start
5. Date production is to be completed
6. Copy one only; four spaces are provided for the machine operator's employee number and the good castings produced.
7. Copy one only; a space is provided for the weigh-counter to record the quantity of good castings transferred to finished goods. A space is also provided for the weigh-counter's signature which specifies that the previous production count agrees closely with the weigh-count check.

The original number one copy of the production order is the shop copy. The second copy of the production order is filed in numerical sequence, by production order number, in the production control

PRODUCTION ORDERS AND MATERIAL REQISITIONS



department. The production order number and suggested production quantity is cross referenced to the perpetual stock control records.

The foreman of the department involved in the first operation on the product distributes the production orders to available operators in his department. At the time of distribution, the foreman and machine operator decide, in the light of the current shop-load situation, and the suggested production control quantity, the exact quantity of raw castings to requisition and machine. When the requisitioned materials arrive, the operator attaches the production order to the box which contains the raw castings. This box is used to move the castings throughout the shop up to the time the castings are transferred into the finished goods inventory. Each time a machine operator completes an order, he places his employee number and the quantity of good castings produced on the appropriate space of the production order form. Production orders are not split, but are maintained in tact throughout the production process.

Foremen of departments other than those involved in the first operation of a product are made aware of the status of the shop and incoming orders by a status report prepared by the production control department every other day.

The company does not maintain tight security over the raw materials storage area. There is no stores clerk and several raw materials handlers have access to the area at all times. Company rules specify that raw material storage is to be considered "off limits" for machine operators at all times. It is doubtful that the policy is very effective as no security fence or other protection devices have been installed.

A machine operator can requisition raw materials by preparing

a material requisition obtained from his department foreman. (See Figure II-2) The raw material requisition is a three-copy, pre-numbered form. It contains spaces for the following information:

1. Type of material requisitioned
2. Quantity requisitioned
3. Quantity of good castings produced
4. Production order number
5. Date
6. Signature of employee
7. Signature of foreman
8. Signature of material handler

The operator completes the three-copy requisition for all of the above information except the quantity of good castings produced, signature of the foreman, and the signature of the material handler. After completing the form with the above information, the operator obtains the approval of the foreman. Upon obtaining approval, he presents the third copy of the requisition to a raw material handler who locates the material and moves it to the proper machine location. The machine operator checks the material located by the material handler for propriety. If agreement is reached between the material handler and the machine operator, the material handler signs all three copies of the material requisition and forwards the third copy to production control. Production control updates the perpetual stock control records for the raw material requisition and files the third copy of the requisition by production order.

If the machine operator completes the production order by the end of the day, he completes the original and second copy of the form

by filling in the actual quantity of good castings produced. He then places both copies of the form in a hopper in the foreman's office. If the machine operator does not complete the production order on the same day the material is requisitioned, he holds the first and second copy of the requisition form until the job is completed. When the production order is complete he fills in the actual production count and disposes of the form in the same manner previously described.

The foreman reviews the material requisition placed in a hopper by the machine operators. His review includes a reasonableness scan of all information included on the form and a verification of the recorded production count. The foreman signifies that he has completed his review by initialing the second copy of the form. He then files the number two copy in numerical sequence, by material requisition number.

The original copies of the material requisitions are collected each morning by a cost accounting clerk. Cost accounting personnel apply predetermined material cost and quantity standards to the requisitions using the standard cost file. Each day the cost accounting personnel batch the costed requisitions by department and file them until the end of the weekly reporting period. At the end of the reporting period the weekly production report and raw material journal entry are prepared from the batches of requisitions processed during the week. The raw materials account is credited at standard cost for the actual quantity of materials requisitioned.

DESCRIPTION OF THE WORK-IN-PROCESS ACCOUNTING AND CONTROL SYSTEM

Work-In-Process Policies

All work in process is carried on the company's records at standard cost. A perpetual inventory of the approximate number of

units-in-process is maintained by the production control department.

Company policy is to cease the production of new orders for one or two days prior to the September 30 physical inventory. This policy permits the producing departments to complete all units in process up to the point of transfer into finished goods. All completed units not physically transferred into finished goods at year end are physically counted and transferred by journal entry to the finished goods inventory account. Perpetual stock control records are adjusted to the physical count.

Work-In-Process Accounting and Control Procedures

All raw materials enter the production process at the beginning of the first productive operation. Accounting and control of raw materials is discussed in the raw materials section.

Direct labor and burden are applied to work-in-process inventory at standard cost for good units produced. Physical direct labor hour standards have been established by time-study and necessarily differ according to the product produced. Direct labor rates vary among the producing departments. A cost build up of the four products is presented as Exhibit II-1.

Indirect labor is charged to overhead along with other manufacturing expenses. Overhead is applied to inventory on the basis of predetermined rates per standard direct labor hour charged to inventory. Burden rates vary depending upon the producing department and the product produced.

Machine operator's labor is handled as direct labor and charged directly to the respective departments. Labor efficiency and rate variances are calculated by operating department and summarized on the weekly

operating report.

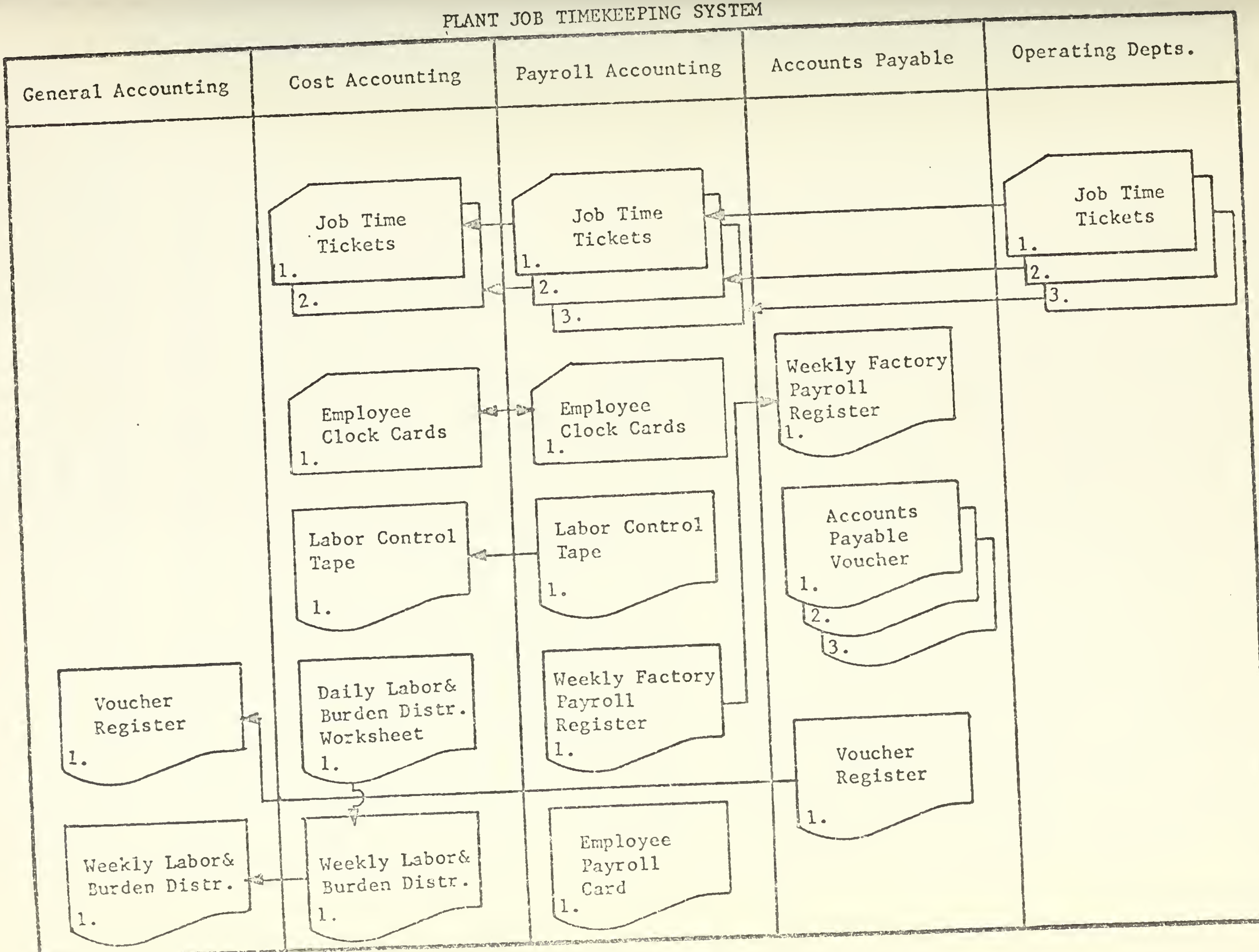
Attendance timekeeping is supervised by the cost accounting department. All factory employees, except supervisory employees, are issued an employee number and are required to clock in and out of the plant. A cost accounting clerk supervises the time clock procedure and delivers the clock cards to the payroll department each morning after the clock-in procedure is completed.

Plant production timekeeping is also supervised by the cost accounting department. Direct labor is reported on job time tickets, which are maintained and prepared by the direct labor employees. Machine operators are the only employees reporting on a direct labor basis. The company's job time ticket is a three-copy form. (See Figure II-3) The form is designed to include spaces for filling in the following information:

1. Employee number (the employee number includes a department number prefix)
2. Job ticket number
3. Time production on the order was started
4. Time production on the order was stopped
5. Total hours worked on the production order
6. Product part number
7. Operation number
8. Good pieces completed
9. Foreman's signature
10. Appropriate spaces for payroll and cost accounting computations

The job ticket number is always the same as the production

PLANT JOB TIMEKEEPING SYSTEM



order number. Multiple production orders cannot be reported on the same job time ticket. Some production orders, however, require more than one day to complete. In such cases multiple job tickets are prepared for the production order. When a production order is begun during a given day and not completed by quitting time, a job time ticket is prepared at the end of the day. An "I" is placed after the job ticket number to signify that the production order is incomplete. Subsequent job time tickets applying to this production order are distinguished by placing a dash after the job ticket number. The dash is followed by a digit which indicates the number of the job time tickets which have been filed on that particular production order. The last ticket of a series applying to a given production order is distinguished by a circle on the job ticket number.

Machine operators prepare a job time ticket for each production order they complete or for production completed on an order they are working on at the close of the day. The machine operator completes the form for the information included in points one, through eight of the above job time ticket description. The operator places the completed form in a hopper located on the foreman's desk. The foreman reviews the job time ticket for propriety and reviews the production count for reasonableness. The foreman signs the job ticket if he finds no exceptions. During the morning of the next business day, a clerk from cost accounting collects the authorized forms from the various department formen and delivers them to the payroll department for processing.

The payroll department sorts the time tickets by employee and compares the total time charged by each employee to the total time noted

on the employee's clock card. Idle time is isolated. Then incentive earnings are computed and noted on each copy of the three copy job time ticket. The payroll clerk separates the third copy of the job time ticket form from the original and second copy. The top two copies are forwarded to cost accounting for further processing. The payroll department batches the third copy of the job time ticket by employee number. Total days earnings and idle time are noted on each batch of job tickets for future use in preparing the factory payroll. Control tapes of total earnings and idle time by department are prepared and used to reconcile to the daily labor distribution worksheet prepared by the cost accounting department.

Cost accounting clerks cost the job tickets for direct labor and burden by referencing the standard cost file.

Standards and extended charges and variances are noted on both the number one and number two copy of the job time ticket. The daily labor and burden distribution worksheet is updated after all the previous days job time tickets have been costed. Labor figures appearing on the distribution are reconciled to the daily labor control tape, prepared by the payroll department. At the end of the weekly reporting period, a weekly labor and burden distribution is prepared from the daily worksheet. The weekly distribution is reconciled to payroll department controls and forwarded to general accounting for posting to the financial records.

The original number one copy of the job ticket is sorted by department and filed by day. The number two copy is filed by job number. All number two copies applying to incomplete production orders are filed by job ticket number in a temporary file.

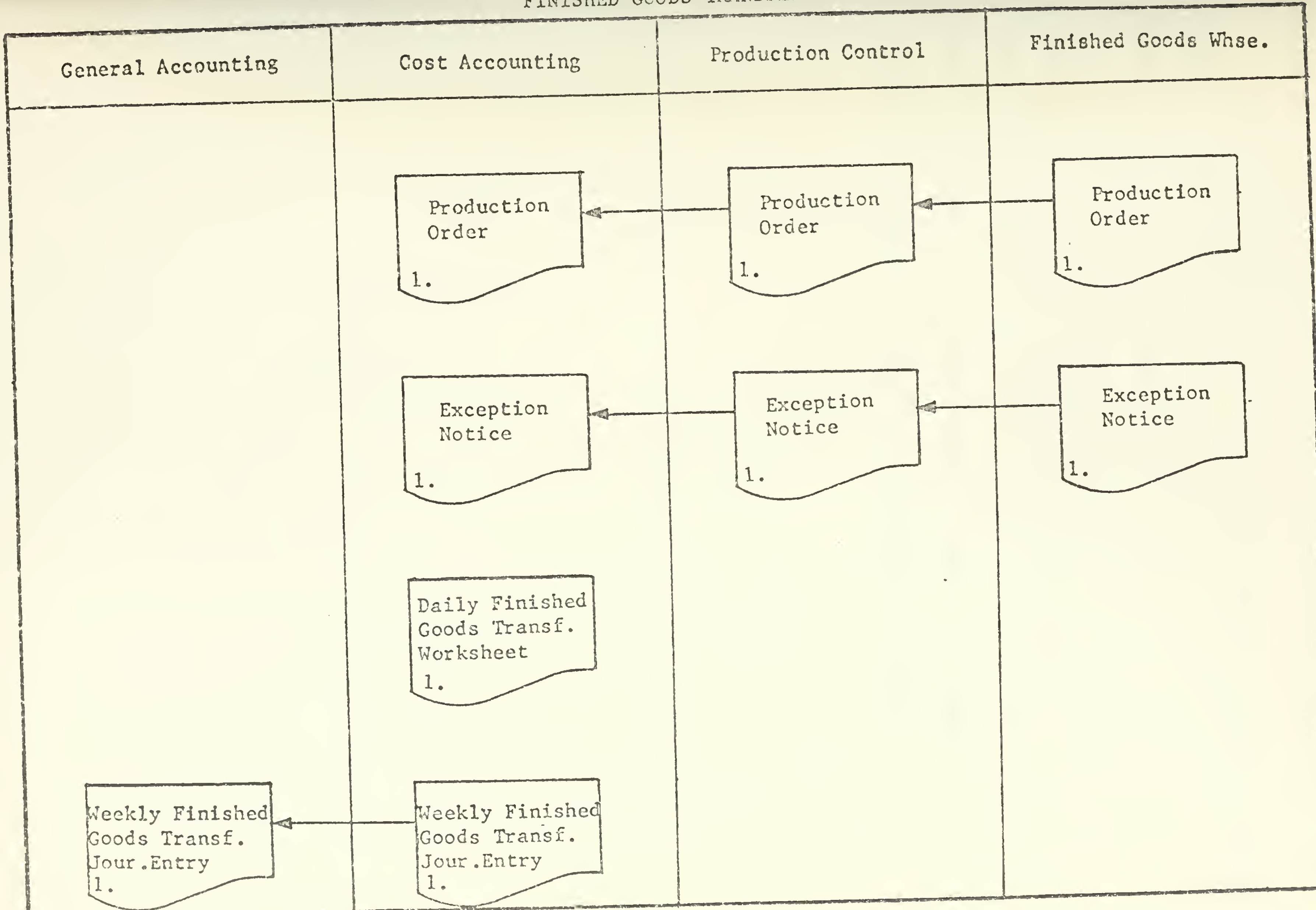
DESCRIPTION OF FINISHED GOODS TRANSFERS ACCOUNTING AND CONTROL SYSTEM; POLICIES AND PROCEDURES

After all operations have been completed on a production order, material handlers move the order to the weigh station located near the finished goods storage area. All production orders must be weigh-counted before transfer to finished goods. The weigh-count serves as a control over production reporting.

The material handler conducting the weigh-count operation verifies the production count on the production order attached to the wooden box containing the finished castings. If the weigh-count differs materially from the production count, the material handler conducting the weigh-count prepares an exception notice which he attaches to the production order. The production order and the attached exception notice are then transferred to production control and the order is set aside until the actual count is ascertained. (See Figure II-4) If no material exception arises as a result of the weigh-count, the material handler signs the production report, dates it, and forwards it to production control. Production control updates the perpetual stock control records by subtracting the production order transferred from the work-in-process file, at the original suggested quantity, and by adding the actual quantity transferred to the finished goods stock control record. All entries to the unit stock control records are cross-referenced to production order numbers.

If an exception notice accompanies the production order, the exception is investigated and resolved before the perpetual records are updated. Employees from production control act as mediators between production foremen, operators, and the employee operating the weigh-station in resolving exception notices. The perpetual file is updated

FINISHED GOODS TRANSFERS



for resolved correction notices in the same manner described for approved finished goods transfers.

After the perpetual records have been updated, production control stamps the production order and correction notices "transferred" and forwards them to the cost accounting department. Cost accounting personnel cost the transfer and prepare a daily transfer worksheet which recaps the day's transfers and exception notices processed. At the end of the weekly reporting period the worksheet is used to prepare the weekly finished goods inventory transfer journal entry. This entry is forwarded to general accounting for posting to the financial records.

EXHIBIT II-2

RAW MATERIALS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF UNITS AND AT STANDARD COST

	Raw Material Number 1	Raw Material Number 2	Raw Material Number 3	Raw Material Number 4
Raw Materials Inventories and Activity; in Terms of Units				
Units in beginning inventory	8,000	8,000	7,000	6,500
Units received during the period	40,000	34,000	34,000	32,000
Units to be accounted for	48,000	42,000	41,000	38,500
Units issued into production	33,600	33,000	33,000	28,900
Units in ending inventory	14,400	9,000	8,000	9,600
Raw Materials Inventories and Activity; at Standard Cost				
Raw Material unit price standard	$ 13.50	$ 16.70	$ 6.50	$ 8.00
Beginning Inventory	108,000	133,600	45,500	52,000
Raw Material received during the period	540,000	567,800	221,000	256,000
Raw Material to be accounted for	$ 648,000	$ 701,400	$ 266,500	$ 308,000
Raw Material issued into production	453,600	551,100	214,500	231,200
Ending inventory	$ 194,400	$ 150,300	$ 52,000	$ 76,800

EXHIBIT II-3

WORK-IN-PROCESS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF UNITS AND AT FULL STANDARD COST

	Product Number 1	Product Number 2	Product Number 3	Product Number 4
Work-In-Process Inventories and Activity In Terms of Units				
Units in beginning inventory	0	0	0	0
Units placed into process during the period	33600	33000	33000	28900
Units to be accounted for	33600	33000	33000	28900
Units transferred to finished goods inventory during the period	30280	30964	29777	21719
Units in ending inventory	3320	2036	3223	7181

EXHIBIT II-3 (Continued)

WORK-IN-PROCESS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF UNITS AND AT FULL STANDARD COST

	Product Number 1	Product Number 2	Product Number 3	Product Number 4	Total for All Four Products
Work-In-Process Inventories and Activity at Standard Cost					
Unit std. cost of product	$ 16.9290	$ 21.8435	$ 9.1900	$ 12.5315	
Beginning inventory	$ 0.00	$ 0.00	$ 0.00	$ 0.00	$ 0.00
Production charges to W.I.P. during the period	568,814.40	720,835.50	303,270.00	362,160.35	1,955,080.25
Total std. cost to be accounted for	$568,814.40	$720,835.50	$303,270.00	$362,160.35	$1,955,080.25
Total std. cost of units transferred to finished goods during the period	512,610.12	676,362.13	273,650.63	272,171.65	1,734,794.53
Ending inventory	$ 56,204.28	$ 44,473.37	$ 29,619.37	$ 89,988.70	$ 220,285.72

EXHIBIT II-4

WORK-IN-PROCESS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF MATERIAL, LABOR AND BURDEN CONTENT

	Product Number 1	Product Number 2	Product Number 3	Product Number 4	Total for All Four Products
Beginning Inventory	$ 0.00	$ 0.00	$ 0.00	$ 0.00	$ 0.00
Direct material charged to W.I.P.	453,600.00	551,100.00	214,500.00	231,200.00	1,450,400.00
Direct labor charged to W.I.P.	20,025.60	29,502.00	15,576.00	22,079.60	87,183.20
Burden applied to W.I.P.	95,188.80	140,233.50	73,194.00	108,880.75	417,497.05
Total charges to W.I.P. for production during the period	$ 568,814.40	$ 720,835.50	$ 303,270.00	$ 362,160.35	$ 1,955,080.25
Direct material content of finished goods transferred	$ 408,780.00	$ 517,098.80	$ 193,550.50	$ 173,752.00	$ 1,293,181.30
Direct labor content of finished goods transferred	18,046.88	27,681.82	14,054.74	16,593.32	76,376.76
Burden content of finished goods transferred	85,783.24	131,581.52	66,045.39	81,826.33	365,236.48
Total std. cost of finished goods transferred	$ 512,610.12	$ 676,362.14	$ 273,650.63	$ 272,171.65	$ 1,734,794.54

EXHIBIT II-4 (Continued)

WORK-IN-PROCESS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF MATERIAL, LABOR AND BURDEN CONTENT

	Product Number 1	Product Number 2	Product Number 3	Product Number 4	Total for All Four Products
Direct material content of ending inventory	$ 44,820.00	$ 34,001.20	$ 20,949.50	$ 57,448.00	$ 157,218.70
Direct labor content of ending inventory	1,978.72	1,820.18	1,521.26	5,486.28	10,806.44
Burden content of ending inventory	9,405.56	8,651.98	7,148.61	27,054.42	52,260.57
Total standard cost of ending inventory	$ 56,204.28	$ 44,473.36	$ 29,619.37	$ 89,988.70	$ 220,285.71

EXHIBIT II-5

FINISHED GOODS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF UNITS AND AT FULL STANDARD COST

	Product Number 1	Product Number 2	Product Number 3	Product Number 4
Finished Goods Inventories and Activity in Terms of Units				
Units in beginning inventory	4,000	4,500	6,000	5,000
Units transferred into finished goods from work-in-process during the period	30,280	30,964	29,777	21,719
Units available for sale during the period	34,280	35,464	35,777	26,719
Units sold and shipped during the period	30,600	31,000	29,000	21,500
Units in ending inventory	3,680	4,464	6,777	5,219

EXHIBIT II-5 (Continued)

FINISHED GOODS INVENTORIES AND ACTIVITY FOR THE NINE MONTH PERIOD ENDED SEPT. 30;

IN TERMS OF UNITS AND AT FULL STANDARD COST

Finished Goods Inventories and Activity at Standard Cost	Product Number 1	Product Number 2	Product Number 3	Product Number 4	Total for All Four Products
Unit standard cost of product	$ 16.9290	$ 21.8435	$ 9.1900	$ 12.5315	
Beginning inventory	$ 67,716.00	$ 98,295.75	$ 55,140.00	$ 62,657.50	$ 283,809.25
Standard cost of units transferred into finished goods from work-in-process during the period	512,610.12	676,362.13	$ 273,650.63	272,171.65	1,734,794.53
Standard cost of units available for sale during the period	$580,326.12	$ 774,657.88	$ 328,790.63	$ 334,829.15	$2,018,603.78
Standard cost of units sold during the period	518,027.40	677,148.50	266,510.00	269,427.25	1,731,113.15
Ending inventory	$ 62,298.72	$ 97,509.38	$ 62,280.63	$ 65,401.90	$ 287,490.63

EXHIBIT II-6

THE MEANS AND STANDARD DEVIATIONS OF THE NORMAL DISTRIBUTIONS USED FOR THE RAW MATERIAL SHIPMENT AND UNITS PRODUCED ON A PRODUCTION ORDER GENERATORS

	Mean	Standard Deviation
Raw Material Shipments:		
Raw Material Number 1	200	25
Raw Material Number 2	180	30
Raw Material Number 3	200	25
Raw Material Number 4	180	30
Units Produced on a Production Order:		
Product Number 1	150	35
Product Number 2	150	35
Product Number 3	150	35
Product Number 4	150	35

EXHIBIT II-8

STANDARDS FOR PRODUCTION COSTING: CORRECT AND ERRONEOUS

	Product Number 1	Product Number 2	Product Number 3	Product Number 4
irect Material				
Correct dir. material std.	$13.5000	$16.7000	$ 6.5000	$ 8.0000
Erroneous dir. material std.	$ 6.5000	$ 8.000	$13.5000	$16.7000
irect Labor				
Correct std. dir. lbr. hrs./unit in Dept. I	.06	.09	.04	.06
Erroneous std. dir. lbr. hrs./unit in Dept. I	.04	.06	.06	.09
Correct std. dir. lbr. hrs./unit in Dept. II	.04	.06	.04	.07
Erroneous std. dir. lbr. hrs./unit in Dept. II	.04	.07	.04	.06
Correct std. dir. lbr. rate in Dept. I	$ 6.20	$ 6.20	$ 6.20	$ 6.20
Erroneous std. dir. lbr. rate Number I in Dept. I	$ 6.00	$ 6.00	$ 6.00	$ 6.00
Erroneous std. dir. lbr. rate Number II in Dept. I	$ 5.60	$ 5.60	$ 5.60	$ 5.60
Correct std. dir. lbr. rate in Dept. II	$ 5.60	$ 5.60	$ 5.60	$ 5.60
Erroneous std. dir. lbr. rate Number I in Dept. II	$ 5.40	$ 5.40	$ 5.40	$ 5.40
Erroneous std. dir. lbr. rate Number II in Dept. II	$ 6.20	$ 6.20	$ 6.20	$ 6.20

This Exhibit is continued on the next page.

EXHIBIT II-8 (Continued)

STANDARDS FOR PRODUCTION COSTING; CORRECT AND ERRONEOUS

	Product Number 1	Product Number 2	Product Number 3	Product Number 4
ec Labor (Continued)				
Correct std. dir. lbr. cost/unit	$ .5960	$ .8940	$ .4720	$ .7640
Erroneous std. dir. lbr. cost/unit	$ .4720	$ .7640	$ .5960	$ .8940
den				
Correct std. burden rates for Dept. I; in dollars/ std. dir. lbr. hr.	$ 12.85	$ 12.85	$ 11.40	$ 11.40
Erroneous std. burden rates for Dept. I; in dollars/ std. dir. lbr. hr.	$ 11.40	$ 11.40	$ 12.85	$ 12.85
Correct std. burden rates for Dept II; in dollars/ std. dir. lbr. hr.	$ 51.55	$ 51.55	$ 44.05	$ 44.05
Erroneous std. burden rates for Dept. II; in dollars/ std. dir. lbr. hr.	$ 44.05	$ 44.05	$ 51.55	$ 51.55
Correct std. burden cost/ unit	$ 2.8330	$ 4.2495	$ 2.2180	$ 3.7675
Erroneous std. burden cost/unit	$ 2.2180	$ 3.7675	$ 2.8330	$ 4.2495
tl Unit Standard Cost				
Correct unit std. cost	$ 16.9290	$21.8435	$ 9.1900	$12.5315
Erroneous unit std. cost	$ 9.1900	$12.5315	$16.9290	$21.8435

EXHIBIT III-1

FREQUENCY OF THE OCCURRENCE OF THE ERRORS

DISCLOSED BY THE AUDIT

Reference Number*	Per System Simulation Model	Per Author's Audit: Per Author's Audit Work Papers	Per Author's Audit: Rounded for the Audit Simulation Model
1. Frequency of occurrence of error in receiving and inspection count operation	25.0%	23.9%	24.0%
2. Frequency of occurrence of error in applying std material price to shipments vouchered	10.0%	8.8%	9.0%
3. Frequency of occurrence of error in reporting production count in Department I	15.0%	13.3%	13.0%
4. Frequency of occurrence of error in reporting production count in Department II	8.0%	8.9%	9.0%
5. Frequency of occurrence of error in applying std dir. lbr. hr. and burden rates to job time tickets	8.0%	6.5%	7.0%
6. Frequency of occurrence of error in applying dir. lbr. rates to job time tickets	10.0%	10.1%	10.0%
7. Frequency of occurrence of error in applying stds. to material requisitions processed	10.0%	10.5%	11.0%

*Reference number refers to the type of error and cross-references to Exhibit III-2 which presents the type of each error referenced on Exhibit III-1.

EXHIBIT III-1 (Continued)

8. Frequency of occurrence of weight-count correction	21 units+	21 units+	21 units+
9. Frequency of occurrence of error in applying stds to production orders transferred	8.0%	9.6%	10.0%

EXHIBIT III-2

TYPE OF THE ERRORS

DISCLOSED BY THE AUDIT

Reference Number*	Per System Simulation Model	Per Author's Audit: Per Author's Audit Work Papers	Per Author's Audit: Rounded for the Audit Simulation Model
1	10% understatement	9.7% understatement	10% understatement
2	Raw Mtl. 1 priced as 3 Raw Mtl. 3 priced as 1 Raw Mtl. 2 priced as 4 Raw Mtl. 4 priced as 2	Raw Mtl. 1 priced as 3 Raw Mtl. 3 priced as 1 Raw Mtl. 2 priced as 4 Raw Mtl. 4 priced as 2	Raw Mtl. 1 priced as 3 Raw Mtl. 3 priced as 1 Raw Mtl. 2 priced as 4 Raw Mtl. 4 priced as 2
3	Prod. orders overstated 10%	Prod. orders overstated 9.7%	Prod. orders overstated 10%
4	Prod. orders overstated 5%	Prod. orders overstated 4.7%	Prod. orders overstated 5%
5	Stds. for Prod. 1 applied to 3 Stds. for Prod. 2 applied to 4 Stds. for Prod. 3 applied to 1 Stds. for Prod. 4 applied to 2	Stds. for Prod. 1 applied to 3 Stds. for Prod. 2 applied to 4 Stds. for Prod. 3 applied to 1 Stds. for Prod. 4 applied to 2	Stds. for Prod. 1 applied to 3 Stds. for Prod. 2 applied to 4 Stds. for Prod. 3 applied to 1 Stds. for Prod. 4 applied to 2
6	Rate for prior year applied 50% of the time Rate for incorrect dept. applied 50% of the time	Rate for prior year applied 51% of the time Rate for incorrect dept. applied 48 % of the time	Rate for prior year applied 52% of the time Rate for incorrect dept. applied for 48% of the time
7	Stds. for Prod. 1 and Mtl. 1 applied to Prod. 3 and Mtl. 3	Stds. for Prod. 1 and Mtl. 1 applied to Prod. 3 and Mtl. 3	Stds. for Prod. 1 and Mtl. 1 applied to Prod. 3 and Mtl. 3

EXHIBIT III-2 (Continued)

TYPE OF THE ERRORS

DISCLOSED BY THE AUDIT

Reference Number*	Per System Simulation Model	Per Author's Audit: Per Author's Audit Work Papers	Per Author's Audit: Rounded for the Audit Simulation Model
7	Stds. for Prod. 2 and Mtl. 2 applied to Prod. 4 and Mtl. 4	Stds. for Prod. 2 and Mtl. 2 applied to Prod. 4 and Mtl. 4	Stds. for Prod. 2 and Mtl. 2 applied to Prod. 4 and Mtl. 4
	Stds. for Prod. 3 and Mtl. 3 applied to Prod. 1 and Mtl. 1	Stds. for Prod. 3 and Mtl. 3 applied to Prod. 1 and Mtl. 1	Stds. for Prod. 3 and Mtl. 3 applied to Prod. 1 and Mtl. 1
	Stds. for Prod. 4 and Mtl. 4 applied to Prod. 2 and Mtl. 2	Stds. for Prod. 4 and Mtl. 4 applied to Prod. 2 and Mtl. 2	Stds. for Prod. 4 and Mtl. 4 applied to Prod. 2 and Mtl. 2
8	N/A	N/A	N/A
9	Stds. for Prod. 1 applied to Prod. 3	Stds. for Prod. 1 applied to Prod. 3	Stds for Prod. 1 applied to Prod. 3
	Stds. for Prod. 2 applied to Prod. 4	Stds. for Prod. 2 applied to Prod. 4	Stds. for Prod. 2 applied to Prod. 4
	Stds. for Prod. 3 applied to Prod. 1	Stds. for Prod. 3 applied to Prod. 1	Stds. for Prod. 3 applied to Prod. 1
	Stds. for Prod. 4 applied to Prod. 2	Stds. for Prod. 4 applied to Prod. 2	Stds. for Prod. 4 applied to Prod. 2

*Reference number refers to the type of error briefly described in Exhibit III-1. For a more detailed description see operating characteristics descriptions in Chapter II.

N/A Not applicable

APPENDIX B

PROGRAM FOR RAW MATERIAL

```
I -OCL-CARD-ENCOUNTERED->#SMC1009-WATFORG (0490,0226,MZ51),'JEESMC1009'
$JOB
      DIMENSION A(5),B(5),N(5)
      REAL LTIN,LTOUT
      DO 1 J=1,5
      TVR=0.
      TAV=0.
      I=1
      READ(5,2)QFREI,QFREO,XMAGI,XMAGO,CFREI,CFREO
    2 FORMAT(2F5.4,2F2.1,2F4.3)
   25 READ(5,3)QIN,QOUT,LTIN,LTOUT,PRC1,PRC2
    3 FORMAT(2F6.0,2F4.0,2F4.1)
      RUNIN=QIN/LTIN
      RUNOUT=QOUT/LTOUT
      Q1IN=XMAGI*LTIN*PRC1
      C1IN=LTIN*(PRC2-PRC1)
      Q1OUT=XMAGO*LTOUT*PRC1
      C1OUT=LTOUT*(PRC2-PRC1)
      VRQI=QFREI*RUNIN
      VRCI=CFREI*RUNIN
      VRQO=QFREO*RUNOUT
      VRCO=CFREO*RUNOUT
      VRIN=VRQI*(Q1IN/270.)**2+VRCI*(C1IN/270.)**2
      VROUT=VRQO*(Q1OUT/270.)**2+VRCO*(C1OUT/270.)**2
      VRT=VRIN+VROUT
      TVR=TVR+VRT
      AVQIN=-Q1IN*VRQI
      AVQOUT=Q1OUT*VRQO
      AVCIN=C1IN*VRCI
      AVCOUT=C1OUT*VRCO
      AVTIN=AVQIN+AVCIN
      AVTOUT=AVQOUT+AVCOUT
      SGMIN=SQRT(VRIN)*270.
      SGMOUT=SQRT(VROUT)*270.
      AVT=AVTIN-AVTOUT
      TSGM=SQRT(VRIN+VROUT)*270.
      TAV=TAV+AVT
      WRITE(6,35)I,AVQIN,AVQOUT,AVCIN,AVCOUT,AVTIN,AVTOUT,SGMIN,SGMOUT
   35 FORMAT(33X,'TAJ #',I2/32X,'***********'//13X,'QIN',F10.2,7X,'QOUT
     1',F10.2//18X,'CIN',F10.2,7X,'COUT',F10.2//18X,'TIN',F10.2,7X,'TO
     2',T, F10.2//16X,'SIGIN',F10.2,5X,'SIGOUT',F10.2)
      WRITE(6,36)AVT,TSGM
   36 FORMAT(29X,'TOTAL AVERAGE',F10.2//29X,'TOTAL STNDVTN',F10.2////)
      I=I+1
      IF(I-5)25,26,26
   26 TSIG=SQRT(TVR)*270.
      WRITE(6,37)TAV,TSIG
   37 FORMAT(/////29X,'TOTAL END RAW'/'-*********************'//30X,'MEAN
     1',F10.2/30X,'STANDARD DEVIATION',F10.2)
      A(J)=TAV
    1 B(J)=TSIG
      WRITE(6,5)
    5 FORMAT(////32X,'* FINAL VALUE *'/33X,'---------------'//30X,'MEAN'
     118X,'STNDEVIATION')
      DO 8 K=1,5
      READ(5,9) N(K)
    9 FORMAT(I3)
    8 WRITE(6,4) N(K),A(K),B(K)
    4 FORMAT(13X,I3,'0',6X,F10.2,15X,F10.2)
      STOP
      END
```

```
$JOB
      DIMENSION R(5),PRO(5),PER(5)
      REAL LHF,LRF
      READ(5,36)QFIN,QFOUT,QMIN,QMOUT,CFIN,CFOUT,LHF,LRF,OHF,QF2,QM2
   36 FORMAT(11F6.3)
      DO 2 I=1,5
      K=1
      TAVE=0
      TQV=0
      TTL=0
      TVL=0
      TTOH=0
      TVOH=0
      TWT=0
      TWTV=0
      TWOUT=0
      TWV=0
      READ(5,4)R(I)
    4 FORMAT(F4.2)
      PRO(I)=100.*R(I)
      WRITE(6,65)PRO(I)
   65 FORMAT(//13X,'***********************************************'//
     1//1X,'RELIABILITY',F4.0,'%')
      WRITE(6,66)
   66 FORMAT(/1X,'PRODUCT#',16X,'RAW',7X,'LABOR',6X,'OH',8X,'WIP',6X,'FI
     1N')
    3 READ(5,37)SP,EP,QIN,QOUT,QW,SCW,ECW,SIZE
   37 FORMAT(2F5.2,3F5.0,2F7.4,F3.0)
      READ(5,38)SLH1,SLH2,ELH1,ELH2,SLR1,SLR2,ELR11,ELR12,ELR21,ELR22,
     1SOH1,SOH2,EOH1,EOH2
   38 FORMAT(10F4.2,4F5.2)
      PER(I)=1.-R(I)
      QXIN=(QIN/SIZE)*QFIN*PER(I)
      QXOUT=(QOUT/150.)*QFOUT*PER(I)
      CXIN=(QIN/SIZE)*CFIN*PER(I)
      CXOUT=(QOUT/150.)*CFOUT*PER(I)
      QLTIN=SIZE*QMIN*SP
      CLTIN=SIZE*(EP-SP)
      QLTOUT=150.*QMOUT*SP
      CLTOUT=150.*(EP-SP)
      QEIN=-QXIN*QLTIN
      QEOUT=QXOUT*QLTOUT
      CEIN=CXIN*CLTIN
      CEOUT=CXOUT*CLTOUT
      TIN=QEIN+CEIN
      TOUT=QEOUT+CEOUT
      QVIN=QXIN*(QLTIN/270.)**2+CXIN*(CLTIN/270.)**2
      QVOUT=QXOUT*(QLTOUT/270.)**2+CXOUT*(CLTOUT/270.)**2
      SIGIN=SQRT(QVIN)*270.
      SIGOUT=SQRT(QVOUT)*270.
      QV=QVIN+QVOUT
      AVE=TIN-TOUT
      STAND=SQRT(QV)*270.
      QL1=150.*QMOUT*SLH1*SLR1
      QX2=(QOUT/150.)*QF2*PER(I)
      QL=150.*QM2*SLH2*SLR2
      QL2=150.*QMOUT*SLH2*SLR2
      QEL=QXOUT*(QL1-QL2)+QX2*QL
      QLV=QXOUT*((QL1/270.)**2+(QL2/270.)**2)+QX2*(QL/270.)**2
      SIGQL=SQRT(QLV)*270.
      HX=(QOUT/150.)*LHF*PER(I)
      HLT1=150.*(ELH1-SLH1)*SLR1
      HLT2=150.*(ELH2-SLH2)*SLR2
      HEL=HX*(HLT1+HLT2)
      VH=HX*((HLT1/270.)**2+(HLT2/270.)**2)
      SIGHL=SQRT(VH)*270.
      RX=(QOUT/150.)*(LRF/2.)*PER(I)
      RLT1=150.*SLH1*(ELR11-SLR1)
      RLT2=150.*SLH1*(ELR21-SLR1)
      RLT3=150.*SLH2*(ELR12-SLR2)
      RLT4=150.*SLH2*(ELR22-SLR2)
      REL=RX*(RLT1+RLT2+RLT3+RLT4)
      VR=RX*((RLT1/270.)**2+(RLT2/270.)**2+(RLT3/270.)**2+(RLT4/270.)*
     1*2)
      SIGR=SQRT(VR)*270.
      TL=QEL+HEL+REL
```

```
      VL=QLV+VH+VR
      SIGL=SQRT(VL)*270.
      OHLT1=150.*QMOUT*SLH1*SOH2
      OHLT2=150.*QMOUT*SLH2*SOH2
      QXOH2=(QOUT/150.)*QF2*PER(I)
      OHLT=150.*QM2*SLH2*SOH2
      QEOH=QXOUT*(OHLT1+OHLT2)+QXOH2*OHLT
      VQOH=QXOUT*((OHLT1/270.)**2+(OHLT2/270.)**2)+QXOH2*(OHLT/270.)**
     12
      SIGQOH=SQRT(VQOH)*270.
      HXOH=(QOUT/150.)*LHF*PER(I)
      OHLTH1=150.*(ELH1-SLH1)*SOH1
      OHLTH2=150.*(ELH2-SLH2)*SOH2
      HEOH=HXOH*(OHLTH1+OHLTH2)
      VHOH=HXOH*((OHLTH1/270.)**2+(OHLTH2/270.)**2)
      SIGHOH=SQRT(VHOH)*270
      RXOH=(QOUT/150.)*CHF*PER(I)
      OHLTR1=150.*SLH1*(EOH1-SOH1)
      OHLTR2=150.*SLH2*(EOH2-SOH2)
      VROH=RXOH*((OHLTR1/270.)**2+(OHLTR2/270.)**2)
      REOH=RXOH*(OHLTR1+OHLTR2)
      SIGROH=SQRT(VROH)*270.
      TOH=QEOH+HEOH+REOH
      VOH=VQOH+VHOH+VROH
      SIGOH=SQRT(VOH)*270.
      WIPIN=TOUT+TL+TOH
      WIPV=QVOUT+VL+VOH
      WIPSIG=SQRT(WIPV)*270.
      QXW1=(QW/150.)*QFOUT*PER(I)
      WLT1=150.*QMOUT*SCW
      QXW2=(QW/150.)*QF2*PER(I)
      WLT2=150.*QM2*SCW
      WQOUT=QXW1*WLT1+QXW2*WLT2
      WQV=QXW1*(WLT1/270.)**2+QXW2*(WLT2/270.)**2
      WQSIG=SQRT(WQV)*270
      CXW=(QW/150.)*QF2*PER(I)
      CLTW=150.*(ECW-SCW)
      WCOUT=CXW*CLTW
      WCV=CXW*(CLTW/270.)**2
      WCSIG=SQRT(WCV)*270.
      WOUT=WQOUT+WCOUT
      WV=WQV+WCV
      WSOUT=SQRT(WV)*270.
      WT=WIPIN-WOUT
      WTV=WIPV+WV
      WTSIG=SQRT(WTV)*270.
      PRO(I)=R(I)*100.
      TAVE=TAVE+AVE
      TQV=TQV+QV
      TTL=TTL+TL
      TVL=TVL+VL
      TTOH=TTOH+TOH
      TVOH=TVOH+VOH
      TWT=TWT+WT
      TWTV=TWTV+WTV
      TWOUT=TWOUT+WOUT
      TWV=TWV+WV
      WRITE(6,67)K,AVE,TL,TOH,WT,WOUT,STAND,SIGL,SIGOH,WTSIG,WSOUT
   67 FORMAT(/5X,I1,7X,'AVE',4X,5F10.2//13X,'SIG',4X,5F10.2)
      K=K+1
      IF(K-5)3,5,5
    5 TSIG=SQRT(TQV)*270.
      TSIGL=SQRT(TVL)*270.
      TSIGOH=SQRT(TVOH)*270.
      TWTSIG=SQRT(TWTV)*270.
      TSWOUT=SQRT(TWV)*270.
      WRITE(6,68)
   68 FORMAT(//30X,'TOTAL ERROR')
      WRITE(6,69)TAVE,TTL,TTOH,TWT,TWOUT,TSIG,TSIGL,TSIGOH,TWTSIG,TSWO
     1UT
   69 FORMAT(//13X,'AVE',4X,5F10.2//13X,'SIG',4X,5F10.2)
    2 CONTINUE
      STOP
      END

$GO
```

VARIABLE - NAME DICTIONARY FOR RAW MATERIAL

PRC1	:	Correct unit cost
PRC2	:	Erroneous unit cost
QFRE1	:	Frequency of input counting error
QFREDØ	:	Frequency of output counting error
XMAGI	:	Magnitude of input counting error
XMAGØ	:	Magnitude of output counting error
CFREI	:	Frequency of input costing error
CFREØ	:	Frequency of output costing error
QIN	:	Number of units of incoming raw material
QØUT	:	Number of units of outgoing raw material (requisition)
LTIN	:	Number of units in one incoming lot
LTØUT	:	Number of units in one outgoing lot
RUNIN	:	Number of lots of incoming raw material
RUNØUT	:	Number of lots of outgoing raw material
Q1IN	:	Dollar value of 1 lot's input counting error
Q1ØUT	:	Dollar value of 1 lot's output counting error
C1IN	:	Dollar value of 1 lot's input costing error
C1ØUT	:	Dollar value of 1 lot's output costing error
VRQI	:	Variance of input counting error in terms of # lot's of input counting error of raw material # 1
VRCI	:	Variance of input counting error
VRQØ	:	Variance of output counting error
VRCØ	:	Variance of output costing error
VRIN	:	Variance of input error
VRØUT	:	Variance of output error

VRT : Total variance = VRIN + VRØUT

TVR : Total accumulated variance = ΣVRT

AVQIN : Mean dollar value of input counting error

AVQØUT : Mean dollar value of output counting error

AVCIN : Mean dollar value of input costing error

AVCØUT : Mean dollar value of output costing error

AVTIN : AVQIN + AVCIN

AVTØUT : AVQOUT + AVCØUT

SGMIN : Dollar value of standard deviation of input error

SGMØUT : Dollar value of standard deviation of output error

AVT : Mean dollar value of error effects on ending raw material

AVT = AVTIN - AVTØUT

TSGM : Dollar value of standard deviation of error effects on ending raw material

TAV : Mean dollar value of cumulated errors

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